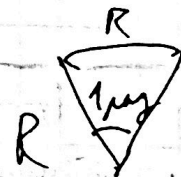
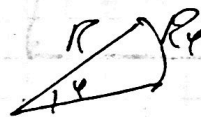
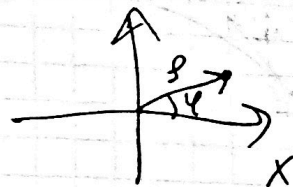


Пример $\Delta X = X_t - X_0$; $x(t), y(t)$; $|\vec{v}| = ?$

$$x = g(t) \cos \varphi(t); \quad \dot{x} = \dot{g} \cos \varphi - g \sin \varphi \cdot \dot{\varphi}$$

$$y = g(t) \sin \varphi(t); \quad \dot{y} = \dot{g} \sin \varphi + g \cos \varphi \cdot \dot{\varphi}$$

$$|\vec{v}|^2 = \dot{g}^2 + g^2 \dot{\varphi}^2$$



$a_c = \text{const}$

$t_{AB} = \tau$ $v_{cp} = ?$

$x(0) = 0$ $|\vec{v}_{cp}| = ?$

$|\vec{a}| = ?$ $t_1 = \frac{\tau}{2}, t_2 = \tau$

$$X_{cp} = \frac{1}{T} \int_0^T X(t) dt$$

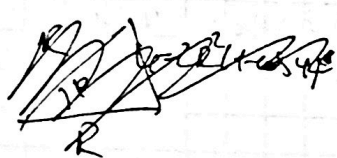
$$x(t) = x_0 + \int_0^t v(t) dt$$

$v(t) = a_c t$; $2\pi R = \int_0^\tau v(t) dt = \int_0^\tau a_c t dt \approx \frac{a_c \tau^2}{2} \Rightarrow a_c = \frac{2\pi R}{\tau^2}$

$$L_{ac} = \frac{a_c (\frac{\tau}{2})^2}{2} = \frac{1}{4} \frac{a_c \cdot \tau^2}{2} = \frac{1}{4} \pi R$$

$$\Rightarrow v_{cp}(t) = \frac{1}{\tau} \int_0^t a_c t dt = \frac{1}{\tau} \frac{a_c t^2}{2} = \frac{a_c t}{2}$$

$$\vec{v}_{cp} = \frac{\vec{v}_{\tau/2}}{2}$$



$$\varphi(t) = \frac{L(t)}{R} = \frac{a_c t^2}{2R}$$

$$|\vec{S}(t)|^2 = 2R^2(1 - \cos \varphi(t)); \quad \vec{a}_{cp} = \frac{\vec{v}_\tau - \vec{v}_0}{\Delta t}; \quad \vec{a}_{cp}(t) = \frac{\vec{v}(t)}{t}$$

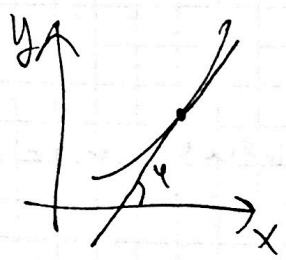
$\boxed{\vec{v}_{cp} = \frac{\vec{S}(t)}{t}}$ $|\vec{v}_{cp}| = \frac{|\vec{S}(t)|}{t}$; $|v_x| = v(t) \cos \varphi, |v_y| = v(t) \sin \varphi$; $|a_x(t)| = \frac{|v_x|}{t}, \dots$

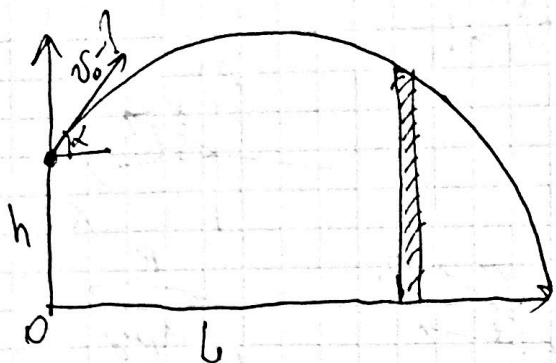
$\vec{v} = b \cdot \vec{i} + c x \vec{j}$
 $a_c = ?, a_n = ?$

$\vec{v} = v_x \vec{i} + v_y \vec{j}; \quad t=0: x_0=0, y_0=0; y(x) = ?$
 $y(t) = \int v_y(t) dt = b c \frac{t^2}{2}; \quad y = \frac{c}{2b} x^2$

$x = x_0 + \int_0^t v_x(t) dt$

$\vec{a} = a_x \vec{i} + a_y \vec{j} = \vec{a}_n + \vec{a}_\tau; \quad |\vec{a}|^2 = a_y^2; \quad a_y = bc, \quad |\vec{a}| = bc$
 $\tan \varphi \approx \frac{dy}{dx} \approx \frac{c}{b} x; \quad a_n = a \cos \varphi$
 $a_\tau = a \sin \varphi \dots$





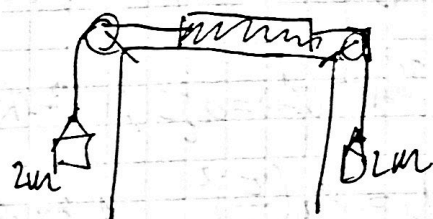
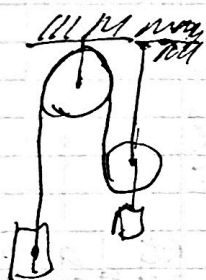
$$\begin{cases} x = v_0 \cos \alpha t \\ y = h + v_0 \sin \alpha t - \frac{gt^2}{2} \end{cases}$$

$$y = h + x \tan \alpha - \frac{g}{2v_0^2 \cos^2 \alpha} x^2$$

$$-(H-h) + x \tan \alpha = \frac{g}{2v_0^2 \cos^2 \alpha} x^2, \quad v_0^2 = \frac{gl^2}{2 \cos^2 \alpha} \cdot \frac{1}{L \tan \alpha + H - h}$$

$$\frac{1 - \cos 2\alpha}{\cos^2 \alpha} = \tan^2 \alpha, \quad \cos^2 \alpha (1 + \tan^2 \alpha)^{-1}$$

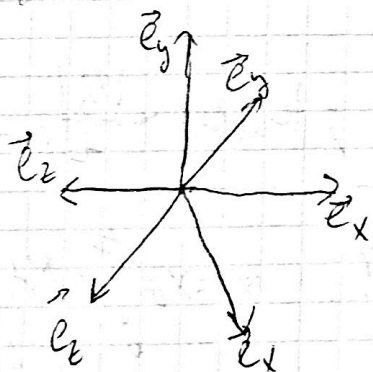
tg монотонна $\Rightarrow$ берём по tg $f'(1); f''(1)$, знак.



Кинематика твёрдого тела

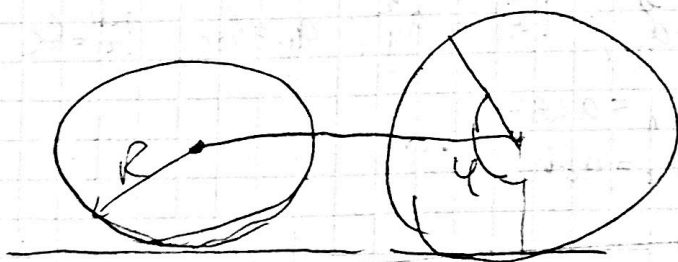
$$\vec{v} = [\vec{\omega} \times \vec{r}]; \quad \vec{\epsilon} = \frac{d\vec{\omega}}{dt}; \quad \vec{a}_\tau = [\vec{\epsilon} \times \vec{r}]$$

Множеств. БСЗ зависят от С.О.



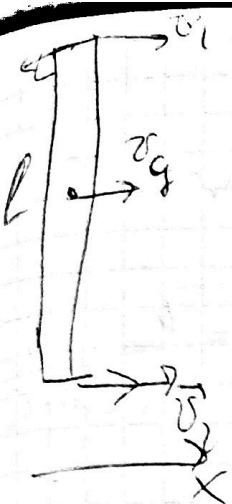
$$\sum_j x_j \vec{e}_j = \vec{r} = \sum_i x'_i \vec{e}'_i \Rightarrow x_i = \sum_j x'_j (\vec{e}'_j, \vec{e}_i)$$

$$\sum_i \sum_j x'_j (\underbrace{(\vec{e}_j, \vec{e}_i)}_{\delta_{ij}}) \vec{e}_i = \sum_j x'_j \cdot \sum_i \delta_{ij} \vec{e}_i = \sum_j x'_j \vec{e}_j$$



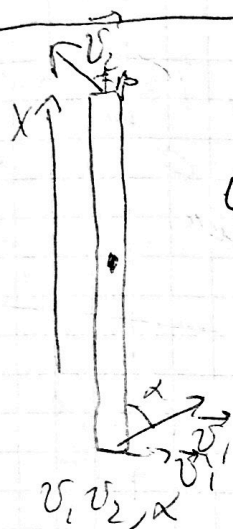
$$\Delta x = L = R\varphi$$

$F_{тр}$ против вып. груза



$$\begin{aligned} \vec{v}_1 &= \vec{v}_y + \vec{v}_1' \\ \vec{v}_2 &= \vec{v}_y + \vec{v}_2' \\ |\vec{v}_1'| &= |\vec{v}_2'| = \omega \frac{l}{2} \\ \vec{v}_1' &= -\vec{v}_2' \Rightarrow \vec{v}_y = \frac{\vec{v}_1 + \vec{v}_2}{2} \end{aligned}$$

$$Ox: v_2 = \frac{v_1 + v_2}{2} + v_2', \quad v_2' = \frac{v_2 - v_1}{2} \Rightarrow v_2' = \frac{\omega l}{2} \Rightarrow \frac{v_2 - v_1}{2}$$



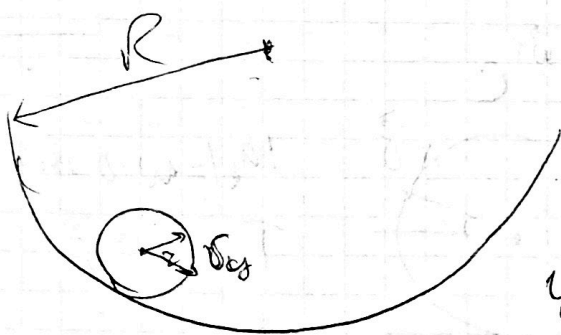
$$\omega = ? \quad \vec{v}_y = \frac{\vec{v}_1 + \vec{v}_2}{2}$$

$$Ox: v_1 \cos \alpha = v_2 \cos \beta = v_y \cdot x \Rightarrow \cos \beta = \frac{v_1 \cos \alpha}{v_2}$$

$$v_y = \frac{v_1 \cos \alpha + v_2 \cos \beta}{2} = v_1 \cos \alpha$$

$$\vec{v}_1' = \vec{v}_1 - \vec{v}_y = v_1 \sin \alpha - v_y y$$

$$Oy: v_{yy} = \frac{v_1 \sin \alpha - v_2 \sin \beta}{2} = \frac{v_1 \sin \alpha - v_2 \sqrt{1 - (\cos \alpha)^2}}{2}$$

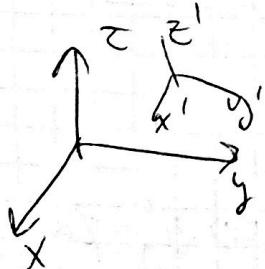


$$R, z, v_c; \omega$$

$$\omega = \dot{\varphi}; \quad l = R \cdot \alpha \Rightarrow \varphi = \frac{\alpha}{2} (R - z); \quad z'(\varphi \alpha)$$

$$\dot{\varphi} = \dot{\alpha} \frac{R - z}{2}; \quad \dot{\alpha} (R - z) = v_y \Rightarrow v_y = R - z.$$

ku. pasera

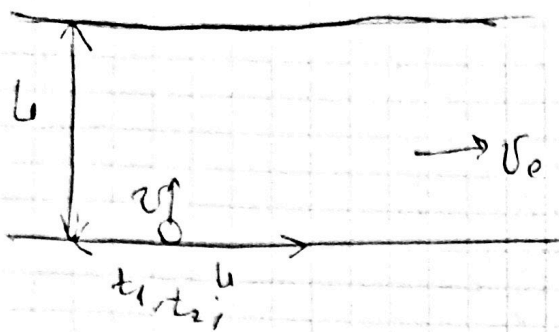


$$\vec{r} = \vec{r}_0 + \vec{r}'; \quad \dot{\vec{r}} = \dot{\vec{r}}_0 + [\vec{\omega} \times \vec{r}']$$

$$\dot{\vec{r}} = \dot{\vec{r}}_0 + \dot{\vec{r}}' + [\vec{\omega} \times \vec{r}']$$

$$\vec{a} = \vec{a}_0 + \sum \ddot{r}_i' \vec{e}_i' + \sum r_i' \ddot{\vec{e}}_i' + 2 \cdot \sum \dot{r}_i' \dot{\vec{e}}_i' + \sum r_i' \ddot{\vec{e}}_i' + \sum r_i' \ddot{\vec{e}}_i' + \sum r_i' \ddot{\vec{e}}_i' + \sum r_i' \ddot{\vec{e}}_i' + \sum r_i' \ddot{\vec{e}}_i' + \sum r_i' \ddot{\vec{e}}_i'$$

repetition. $-\omega^2 \vec{r}_i$



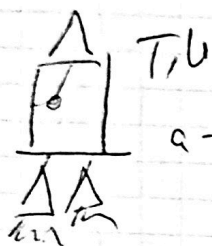
$$\alpha = \frac{u}{v_0} - ? \quad \vec{v} = \vec{u} + \vec{v}_0$$

$$0 = u_x + v_{0x} \Rightarrow u_x = -v_0$$

$$\text{OY: } v = v_{y1}$$

$$L_1 = \frac{2L}{u_y}, \quad t_2 = \frac{L}{u+v_0} + \frac{L}{u-v_0}$$

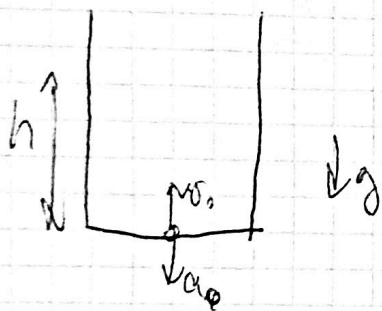
$$u^2 = u_y^2 + v_0^2; \quad u_y^2 = u^2 - v_0^2$$



$$\vec{a} = \vec{a}_0 + \vec{a}'; \quad g = -a + g'; \quad g' = a + g; \quad \omega^2 = \frac{g'}{L}, \quad T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g'}}$$

$$m\vec{a}' = m\vec{a}, \quad m\vec{a} = -mg + m\vec{a}' \approx m\vec{a}', \quad m\vec{a}' + \frac{L}{r} \omega^2 \vec{r} = 0$$

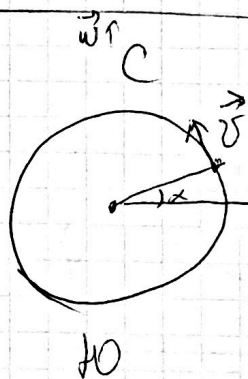
$$\omega = \omega^2$$



$$\vec{a} = \vec{a}_0 + \vec{a}', \quad \vec{a}' = \vec{a} - \vec{a}_0$$

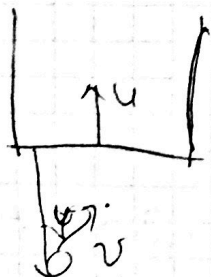
$$\text{OY: } a' = g - a_0; \quad \Delta y = \frac{v_0^2 - v_0^2}{2a'}$$

$$|a_{y,c}| = \omega^2 R; \quad \omega = 2\pi n$$



$$|a_c| = \omega \cdot v \cdot \sin \varphi$$

$$T, \omega = \frac{2\pi}{T}$$



$$v_1 - ?$$

$$\text{C.O. number; } \vec{v} = \vec{u} + \vec{v}'$$

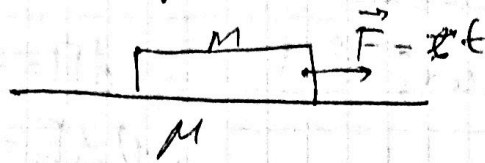
$$\text{OX: } v \sin \varphi = v_0 + v'_x$$

$$\text{OY: } v \cos \varphi = u + v'_y, \Rightarrow v'_y = v \cos \varphi - u$$

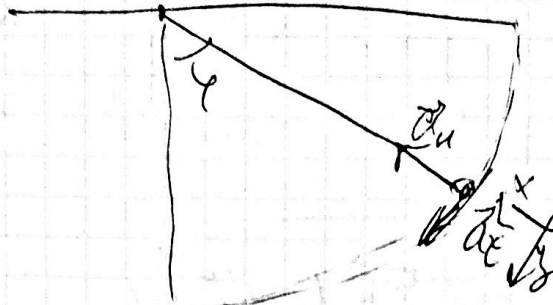
$$v_{y1} = u - v \cos \varphi + u = 2u - v \cos \varphi$$

$$F_{kl} = G \frac{q_1 q_2}{r_{12}^2} \frac{\vec{r}_{12}}{r_{12}}$$

Горизонт



1) ~~поиск~~ ~~уравнения~~ ~~движения~~
2) ~~поиск~~ ~~уравнения~~ ~~движения~~

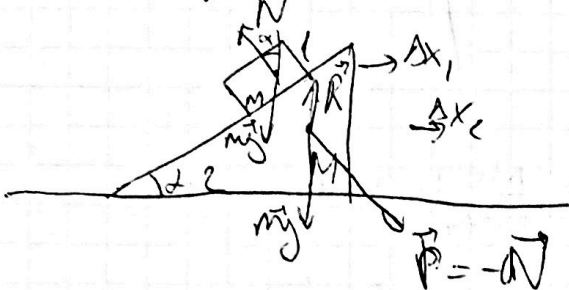


- 1) $a(\varphi), T(\varphi)$
- 2) $T(v_y = v_{y_{max}})$

$\vec{a} = \vec{a}_n + \vec{a}_\tau$; $0, T = mg \cos \varphi$
 $\delta A = (\vec{F}, d\vec{s}) = 0 \Rightarrow \{C\}$
 $0 = \frac{mv^2}{2} - mgl \cos \varphi$; $v^2 = 2gl \cos \varphi$; $a_n = \frac{v^2}{r} \Rightarrow a_n = 2g \cos \varphi$
 $0, T = mg \sin \varphi = ma_\tau \Rightarrow a_\tau = g \sin \varphi$

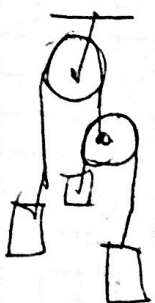
$T = 3mg \cos \varphi$

$v = \sqrt{v^2}$, $v_y = v \sin \varphi$ (1', 1'')



$\uparrow x$: $0, -N \sin \alpha = m \ddot{x}_1$
 $\downarrow y$: $0, mg - N \cos \alpha = m \ddot{y}_1$
 $0, Mg + P \cos \alpha - R = 0$
 $0, P \sin \alpha = M \ddot{x}_2$

$\delta(x_2 - x_1) = \delta y_1$

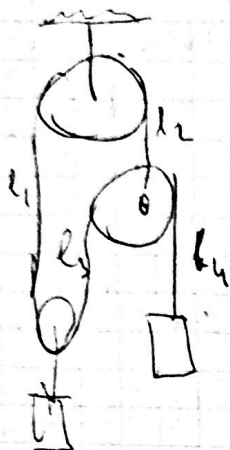


3 m. 2 T

Δx_1
 $\Delta x_2 = -\Delta x_1 + \Delta x_3 \delta \Rightarrow \Delta x_2 + \Delta x_3 = -\Delta x_1$
 $\Delta x_3 = -\Delta x_1 - \Delta x_2 \delta$

Задача

Рассмотрим l_1 !



$$d\Pi = -\delta A_n$$

$$1) \Delta E_k = A$$

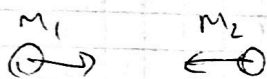
$$2) \Delta E_k + \Delta E_n = 0$$

$$M \vec{a}_{\text{ц.м.}} = \sum m_i \vec{a}_i = \sum \vec{F}_i = \sum \vec{F}_{i, \text{вн.}} + \sum \vec{F}_{i, \text{внутр.}}$$

$$\vec{P}_{\text{вн.}} = 0 \text{ (если нет внешних сил);}$$

Центр масс - и ц.м.
внутр. и внеш. силы

Центр масс - и ц.м.
внутр. и внеш. силы



абсолютно упруг.
3CU
3CD

абсолютно упруг.

$$m_1 \vec{v}_1 + m_2 \vec{v}_2 = m_1 \vec{v}_1' + m_2 \vec{v}_2'$$

$$\frac{m_1 v_1^2}{2} + \frac{m_2 v_2^2}{2} = \frac{m_1 v_1'^2}{2} + \frac{m_2 v_2'^2}{2}$$

абсолютно упруг. - центр

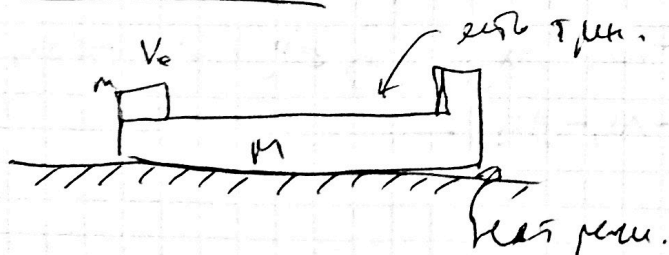
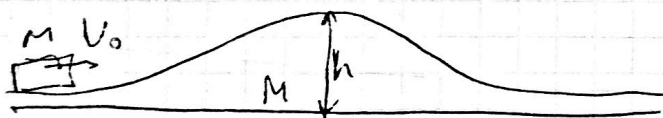
$$m_1 \vec{v}_1 + m_2 \vec{v}_2 = m_1 \vec{v}_1' + m_2 \vec{v}_2'$$

$$\vec{v} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2}{m_1 + m_2}$$

$$+ Q$$

$$\frac{kx^2}{2} = A_{\text{упр.}}$$

$$\text{Б.С.О.У.С.} \therefore v_1' = \frac{m_2}{m_1 + m_2} (v_2 + v_1) \quad v_2' = -\frac{m_1}{m_1 + m_2} (v_2 + v_1)$$



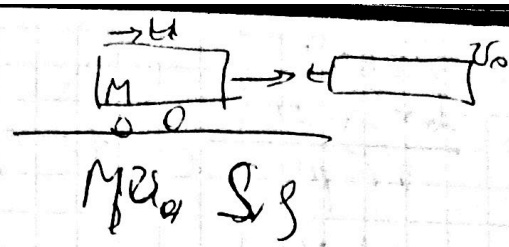
$$\text{или: } V = \frac{mv_0}{m+M} ; E_{k1} = \frac{Mv_0^2}{2}$$

$$E_{k2} = (m+M) \frac{v^2}{2}$$

$$A = -\mu mg 2L$$

$$\Delta E_k = A$$

$$\Delta E = E_{v_2} - E_{v_1}$$



$$u(t) = ?$$

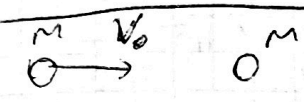
$$dx: Mu + dm v_0$$

$$(M+dm)(u+du) = Mu + Mdu + dm u + dm du$$

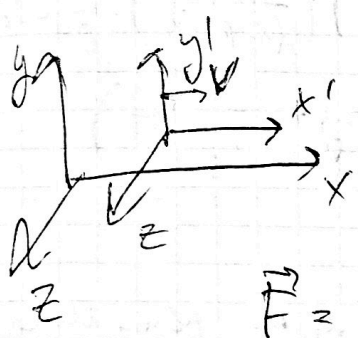
$$Mdu = -dm v_0 - dm u$$

$$dm = \rho dV = \rho \cdot S \cdot dl = \rho S (v_0 + u) dt \Rightarrow M \frac{du}{dt} = -(v_0 + u)^2 S$$

$$\Rightarrow \ln \frac{1}{u+v_0} = \frac{\rho S}{M} t$$



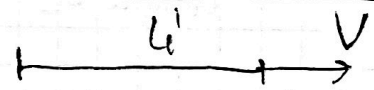
Линейка



$$x = \frac{x' + vt'}{\sqrt{1-\beta^2}}, y = y', z = z'; \beta = \frac{v}{c}$$

$$t = \frac{t' + \frac{v}{c^2} x'}{\sqrt{1-\beta^2}}; x' = \frac{x - vt}{\sqrt{1-\beta^2}}, t' = \frac{t - \frac{v}{c^2} x}{\sqrt{1-\beta^2}}$$

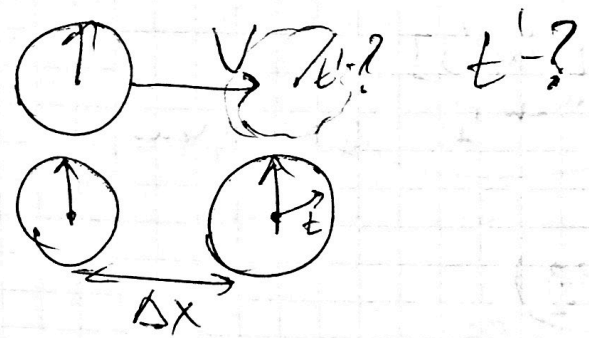
$$\vec{F} = \dot{\vec{p}}, \vec{p} = \frac{m\vec{v}}{\sqrt{1-\beta^2}}; E = \frac{mc^2}{\sqrt{1-\beta^2}}, E^2 = (mc^2)^2 + p^2 c^2$$



$$l = x_2^H - x_1^H \text{ в один момент времени}$$

$$l' = x_2'(t_2') - x_1'(t_1') = \frac{(x_2 - vt) - vt}{\sqrt{1-\beta^2}} - \frac{(x_1 - vt) - vt}{\sqrt{1-\beta^2}} = \frac{l}{\sqrt{1-\beta^2}}$$

оно падает;
т.к. в это же время
в координатах



$$\Delta x = vt \Rightarrow t = \frac{l'}{v} \Rightarrow \Delta t = \frac{l'}{v} \Rightarrow \Delta t = \frac{l'}{v} \Rightarrow \Delta t = \frac{l'}{v} \Rightarrow \Delta t = \frac{l'}{v}$$

$$= l' \sqrt{1-\beta^2}$$

Выразим через координаты:

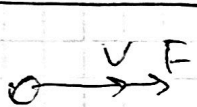
$$V_x = \frac{dx}{dt} ; dx = \frac{dx' - v dt'}{\sqrt{1-\beta^2}} , dt = \frac{dt' + dx' \frac{v}{c^2}}{\sqrt{1-\beta^2}}$$

$$\frac{dx' + v dt'}{dt' + dx' \frac{v}{c^2}} = \frac{v' + v}{1 + v_x \frac{v}{c^2}}$$

$v = c \cdot c.o.$

$v_x = c$ - бегущая волна

$$v_y = \frac{dy}{dt} = \frac{dy'}{dt} = \frac{dy'}{\frac{dt' + dx' \frac{v}{c^2}}{\sqrt{1-\beta^2}}} = \frac{v'}{1 + \frac{v'_x v}{c^2}} \sqrt{1-\beta^2}$$



а-?

$$F_x^{max} = \dot{p}_x = \frac{dp_x}{dt}$$

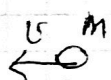
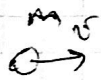
$$p_x = \frac{mv_x}{\sqrt{1-\frac{v^2}{c^2}}} ; \dot{p}_x = \frac{m \dot{v}_x}{\sqrt{1-\beta^2}} + m v_x \cdot \left(+\frac{1}{2}\right) \left(\frac{1}{(1-\beta^2)^{3/2}}\right) \cdot \frac{1}{c^2} 2 v_x \cdot a_x$$

$$= \max \left(\frac{1}{\sqrt{1-\beta^2}} + \frac{v_x^2}{(1-\beta^2)^{3/2}} \right)$$

$$p_x = \frac{mv_x}{\sqrt{1-\frac{v^2}{c^2}}}$$

где $\dot{v}_x$:

Среднее значение



Рассмотрим м. в замкнутой системе. У нас перемещение и пер. импульс сохранены

$$p_{total} = m v - m v = m v_k = 0$$

$$E = \frac{2mc^2}{\sqrt{1-\beta^2}} = Mc^2 \quad (\beta = \cos \theta = 0)$$

$$M = \frac{2m}{\sqrt{1-\beta^2}} > 2m$$

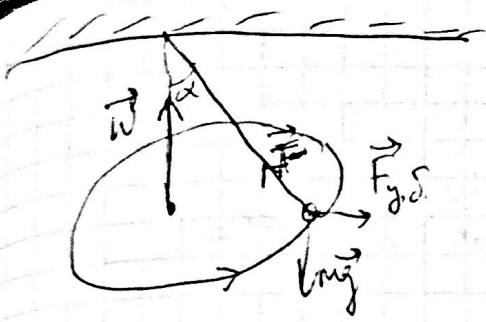
Лемма

$$\vec{F} = m \vec{a} , \quad \vec{a} = \vec{a}_0 + \vec{a}' + 2[\vec{\omega} \times \vec{v}'] + [\vec{\omega} \times [\vec{\omega} \times \vec{r}']]$$

и т.д.

$$m \vec{a}' = \vec{F} + \vec{F}_{in} \quad \text{не имеет вес. знач.},$$

$$\vec{F}_{in} = m(\vec{a}' - \vec{a}) = m(-\vec{a}_0 - 2[\vec{\omega} \times \vec{v}'] + \underbrace{[\vec{\omega} \times [\vec{\omega} \times \vec{r}']]}_{\text{гиротерм}})$$



$$\Delta x = 2l_0 - l_0 \approx l_0 : T = f_0$$

$$k = \frac{f_0}{\Delta x} = \frac{f_0}{l_0}$$

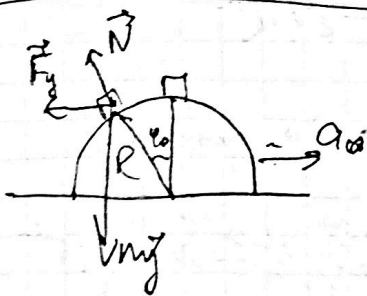
$\alpha - ?$
 $l - ?$

$$T = k(l - l_0) = f_0 \frac{(l - l_0)}{l_0}$$

$$\vec{F}_{y0} + m\vec{g} + \vec{T} = m\vec{a}' = 0$$

$$Ox: m\omega^2 r_{\perp} - T \sin \alpha = 0$$

$$Oy: m\vec{g}$$



$$\vec{F}_{un} + m\vec{g} + \vec{N} = m\vec{a}'$$

$$a_n = \frac{v^2}{R}$$

$$Ox: mg \cos \phi - m a_n \sin \phi = m a_n = m \frac{v^2}{R}$$

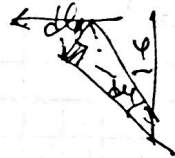
Work on mass: ΔK

$$\Delta K = \sum A = A_{th} + A_{ng} ; A_{ng} = mg(1 - \cos \phi_0)$$

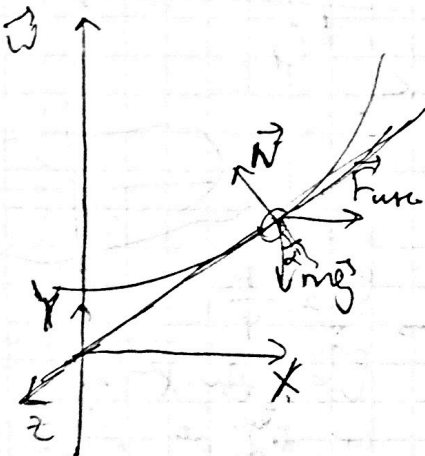
cause
person been run

$$\delta A_{un} = (\vec{F}_{un} \cdot d\vec{r}) = m a_n R d\phi \cos \phi$$

$$\int_0^{\phi_0} \delta A_{un} = m a_n R \sin \phi_0$$



$\Rightarrow$ check using v at $\phi_0 \Rightarrow$
 $\Rightarrow$ same.



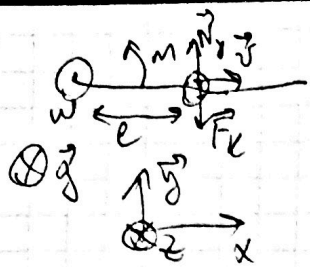
$$\vec{F}_{y0} + m\vec{g} + \vec{N} = 0$$

$$Ox: mg \sin \alpha - m\omega^2 r_{\perp} \cos \alpha = 0$$

$$\tan \alpha = \frac{x\omega^2}{g}$$

$$\tan \alpha = \frac{dy}{dx} , \frac{dy}{dx} = \frac{\omega^2}{g} x , y = \frac{\omega^2}{2g} x^2 + C$$





$$N=1 \quad N_z = mg, \quad \vec{F}_k = 2\epsilon \vec{\omega} \times \vec{v}$$

$$\vec{F}_{gs} = m\omega^2 \vec{r}$$

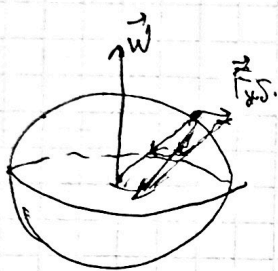
$$Ox: m\ddot{x} = m\omega^2 x$$

$$\ddot{x} = \dot{v}_x = \frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt} = \frac{dv}{dx} v_x = v_x$$

$$\frac{dv_x}{dx} v_x = \omega^2 x$$

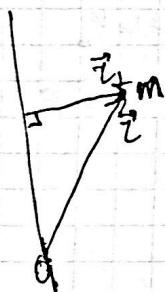
$$v_x dv_x = \omega^2 x dx, \quad v_x^2 = \omega^2 x^2 + C, \quad r(x=0) = v_0$$

$$v_x^2 = \omega^2 x^2 + v_0^2$$



New Seminar

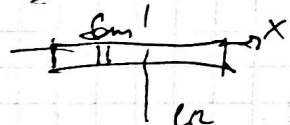
$$J = m r_{\perp}^2, \quad J = \sum m_i r_{\perp i}^2$$



$$J = \int dm(r) \cdot r_{\perp}^2 = \int g(r) r_{\perp}^2 dV$$

$$J_c = \int g(r) r^2 dV$$

$$E_k = \frac{mv^2}{2} = \frac{J\omega^2}{2}$$



$$J = \int dJ$$

$$dJ = dm x^2$$

$$J_c = \int_{-l/2}^{l/2} dm x^2 = \int_{-l/2}^{l/2} \frac{M}{l} dx x^2 = \left(\frac{M}{l} \frac{x^3}{3} \right) \Big|_{-l/2}^{l/2} =$$

$$= \frac{1}{12} M l^2, \quad J_y = J_c + m a^2 = \left(\frac{1}{12} + \frac{1}{4} \right) M l^2 = \frac{1}{3} M l^2$$



$$dm = dr dz d\phi \cdot \sigma$$

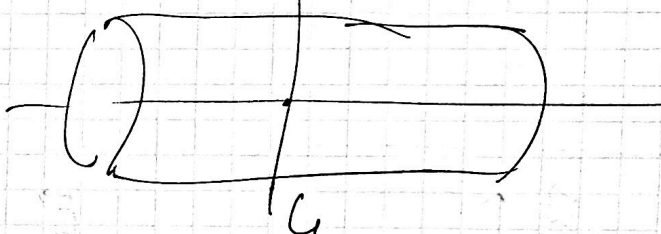
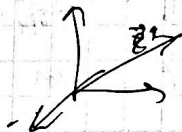
$$J = \int dJ = \int \int \sigma \cdot r^2 dr d\phi \cdot z^2$$

$$= 2\pi \frac{M}{\pi R^2 h} \int_0^R r^3 dr = \frac{1}{2} M R^2$$

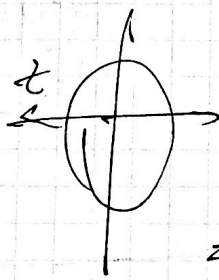
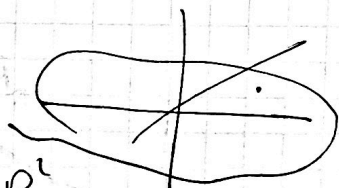
Die ① ist gegeben.

$$J_x + J_y + J_z = 2 \int dm r^2 = 2 \int dm r^2$$

statische Mom. ②



$$J = \frac{m R^2}{2}$$



$$J_y = \frac{m R^2}{4}$$

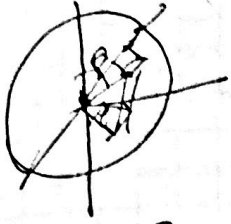
$$J = \int dm x^2$$

$$= \int_{-R}^R \frac{dm}{4} x^2 + dm x^2 \quad \text{③}$$

$$dm = g dV = \frac{M}{\pi R^2 h} \pi R^2 dx$$

$$\text{③} \frac{M}{4} \int_{-R}^R x^2 + x^2 dx =$$

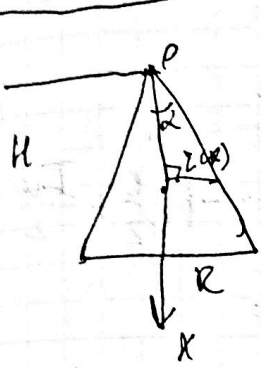
$$dV = r^2 \sin \theta dr d\theta d\phi$$



$$I = \int r^2 \sin^2 \theta d\theta d\phi$$

$$I = \int_0^{\pi} \int_0^{2\pi} r^2 \sin^2 \theta d\theta d\phi$$

$$I = \frac{2}{5} MR^2$$



$$z_c = \frac{1}{M} \int z dM$$

$$x_c = \frac{1}{M} \int x dM$$

$$dV = \rho(x) dx = (x \tan \alpha)^2 dx$$

$$\rho = \frac{M}{\int dV} = \frac{M}{\int (x \tan \alpha)^2 dx}$$

$$x_c = \frac{1}{M} \int \frac{M}{\int (x \tan \alpha)^2 dx} x (x \tan \alpha)^2 dx = \frac{3}{4} H$$

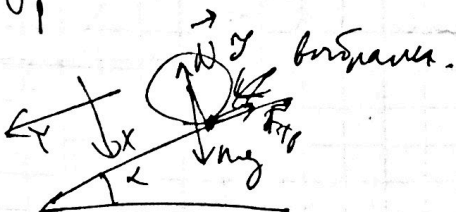
Динамика Твёрдого тела

$$\sum \vec{F}_i = m \vec{a}_c, \quad \vec{M} = \sum (\vec{r}_i \times \vec{F}_i)$$

$$\vec{M}_n = \sum (\vec{r}_{i,n} \times \vec{F}_i)$$

$$J \vec{\epsilon} = \vec{M}_n, \quad K = \frac{m v_c^2}{2} + \frac{J \omega^2}{2}$$

$$\vec{N}_n = \sum \vec{r}_i \times m \vec{v}_{i,n}$$



$$\vec{F}_{tp} + \vec{N} + m\vec{g} = m\vec{a}_c, \quad \vec{M}_{T_{ax}} = I \vec{\epsilon}$$

$$Ox: \begin{cases} mg \cos \alpha - N = 0 \end{cases} \text{ где } \begin{matrix} \text{норм.} \\ \text{сил.} \end{matrix}$$

$$Oy: \begin{cases} mg \sin \alpha - F_{tp} = ma_c \end{cases}$$

$$mg \sin \alpha = I \epsilon = J \frac{a_c}{R}$$

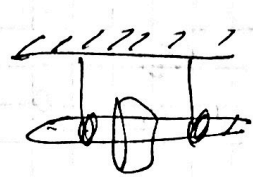
$$y = R \cdot \epsilon, \quad a = \epsilon \cdot R$$

$$\Rightarrow a = \frac{2}{3} g \sin \alpha$$

$$F_{tp} = \frac{1}{3} mg \sin \alpha$$

$$J = J_o + m R^2$$

$$J_o = \frac{m R^2}{2}, \quad J = \frac{3}{2} J_o$$



$$Ox: (M + m) a = (M + m) g - T$$

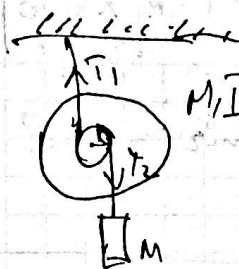


$$T \cdot R = J \epsilon$$

$$2 \epsilon = a, \quad y = \frac{m v_c^2}{2}$$

$$J' = \frac{m v_c^2}{2} + \frac{4 M (R^2 + 2 r^2)}{2}$$

$$y = \int v dv$$

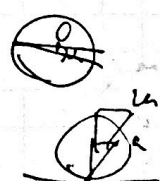


$$M g - T_1 + T_2 = M a_1$$

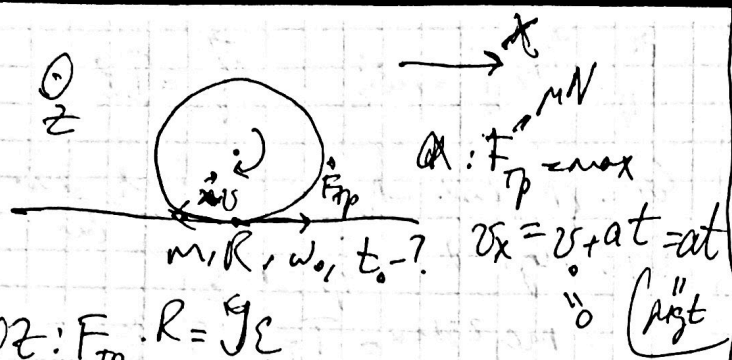
$$m g - T_2 = m a_2$$

$$(T_1 \cdot R + T_2) R = J \epsilon$$

$$a_1 = R \epsilon, \quad a_2 = 2 a_1$$



②



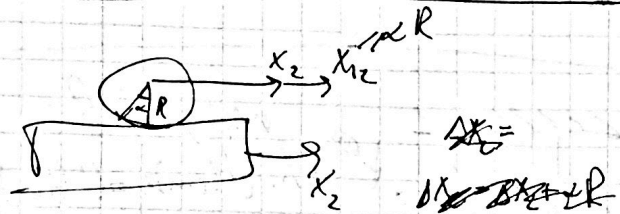
А: $F_{fp} = \max$

$v_x = v + at = at$
 $\omega = \omega_0 + \epsilon t$

от: $F_{fp} \cdot R = G \epsilon$

$\epsilon = \frac{F_{fp} \cdot R}{g}$, $\omega(t) = -\omega_0 + \epsilon t$

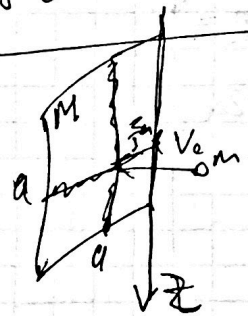
Прокен. зачен-а, кога $\omega(t) = -\omega_0 + \epsilon t$
 $= \omega(t) \cdot R$



Нен-Сендер

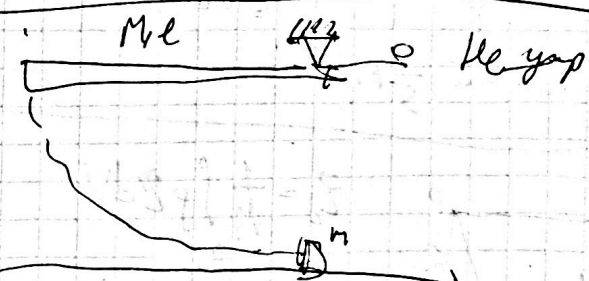
$\vec{N}_u = G \vec{\omega}$ - an-no neng. sen.
 Емн главно $\vec{N}_u = [\vec{e}_1, \vec{p}_c] + G \vec{\omega}$
 $\frac{d\vec{N}}{dt} = \frac{1}{2m} \vec{N}$

$z = z(t)$, $\varphi = \varphi(t)$, $\vec{N} = ?$
 $\vec{N} = [\vec{z}, m\vec{v}] = m \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x & y & 0 \\ \dot{x} & \dot{y} & 0 \end{vmatrix} = m \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x & y & 0 \\ \dot{x} & \dot{y} & 0 \end{vmatrix}$
 $\vec{v}_x = \dot{x} = r \cos \varphi - z \dot{\varphi} \sin \varphi$
 $\vec{v}_y = \dot{y} = z \sin \varphi + r \dot{\varphi} \cos \varphi$



пронек.
 от: $\vec{N}_u = \frac{2}{3} am v_0$
 $\vec{N}_k = \frac{2}{3} am v_1 + G \vec{\omega}$
 $\vec{v} = \left(\frac{1}{12} + \frac{1}{2} \right) m a^2$

$E_u = \frac{mv_u^2}{2}$, $E_k = \frac{mv_k^2}{2} + \frac{J \omega^2}{2}$
 $\frac{mv_u^2}{2} + \frac{J \omega^2}{2}$



1) $0 = -Mg \frac{l}{2} + \frac{I \omega^2}{2}$

$I = \frac{1}{3} M l^2$ $\omega^2 = \frac{3g}{l}$

2) $\vec{v}$ direction moment

2) $J_u = J_k + m v \frac{v}{\omega}$
 $\vec{v} \cdot \vec{\omega}$

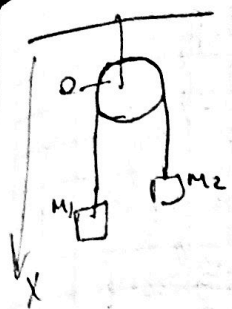
Нен-Сендер

~~Non-Sender~~
 $\vec{p} = \frac{\partial L}{\partial \dot{q}_i}$
 $\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = Q_i$
 $\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = 0$

$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = 0$

$K = K(q, \dot{q})$, $\Pi = \Pi(q)$

кае згоно K и Π , акае
 впрашен зго q_i



$m_1, m_2; I, R$

$x_1, x_2; x_1 + x_2 + \pi R = l = \text{const}$

$l > 0$, when $x_1 = 0$;

$x_1 = R\varphi$;

$$U = -m_1 g x_1 - m_2 g x_2, \quad K = \frac{m_1 \dot{x}_1^2}{2} + \frac{m_2 \dot{x}_2^2}{2} +$$

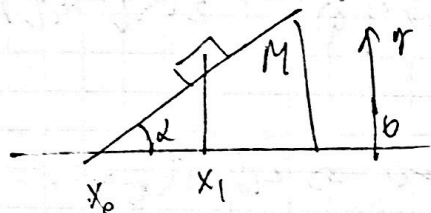
$$U = -g(m_1 R\varphi + m_2(l - \pi R - R\varphi)) + \frac{I \dot{\varphi}^2}{2}$$

$\varphi \cdot g R(m_2 - m_1) + C_2$

$$K = \left(\frac{m_1 R^2}{2} + \frac{m_2 R^2}{2} + \frac{I}{2} \right) \dot{\varphi}^2 - \frac{\varphi^2}{2} (m_1 + m_2) R^2 + I$$

$$L = K - U$$

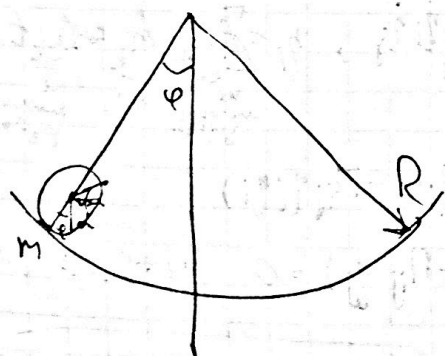
$$\text{Ур-е Ла-гранжа } \ddot{\varphi}((m_1 + m_2)R^2 + I) + (m_2 - m_1)Rl = 0$$



$$(x_1 - x_0) \cdot \tan \alpha = y_1,$$

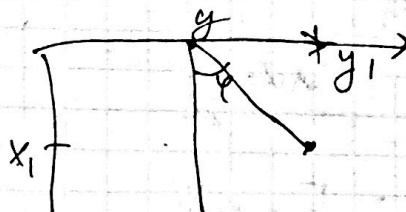
$$U = U_{\text{kin}} + mgy, \quad K = \frac{M \dot{x}_0^2}{2} + \frac{M (\dot{x}_1^2 + \dot{y}_1^2)}{2}$$

$L = U, \text{ const.}$



$$(\varphi + \alpha)R = r\varphi$$

$$U = -mg(R - r)\cos \varphi, \quad K = \frac{mv_0^2}{2} + \frac{I\omega^2}{2}; \quad v_0 = \dot{\varphi}(R - r)$$



$$y = R \cos \omega t$$

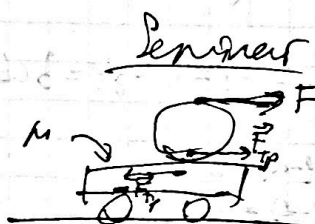
y_1 — ось y1

$$(y - y_1)^2 + x_1^2 = l^2;$$

$$x_1 = l \cos \varphi \Rightarrow U = -mgx_1 = -mg(l \cos \varphi)$$

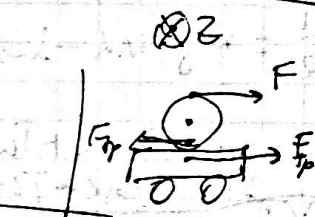
$$y_1 = y + l \sin \varphi \Rightarrow K = \frac{m}{2} (\dot{x}_1^2 + \dot{y}_1^2)$$

Одна ось y1 — ось y1.



$$F + F_{tp} = ma_1$$

$$-F_{tp} = -Ma_2$$



$$F - F_{tp} = ma_1$$

$$F_{tp} = Ma_2$$

$$R(F + F_{tp}) = J\epsilon$$

$$\Delta x_1 = \Delta x_2 + \varphi R; \quad a_1 = a_2 + \epsilon R$$

$$m\epsilon R = m(a_1 - a_2),$$

$$\frac{mR^2(F + F_{tp})}{J} = F - F_{tp} \left(1 + \frac{m}{M} \right)$$

$$F_{tp} (mR^2 + J(1 + \frac{m}{M})) = F(J - mR^2) \leq mR^2$$

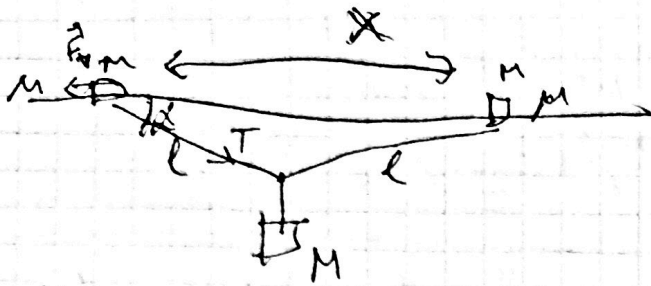
Статика равновесия

механика.

Равновесие: $\sum \vec{F}_i = 0$

$\frac{\partial U}{\partial q_i} = 0$ one been i.

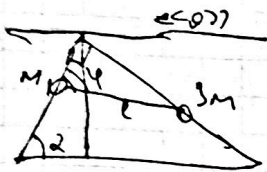
$-Q_i$



$$Mg - 2T \sin \alpha = 0; \cos \alpha = \frac{x}{2l}$$

$$T \cos \alpha + mg - N = 0$$

$$-F_{\text{fr}} + T \sin \alpha = 0; F_{\text{fr}} = \mu N$$



$T = ?; \phi = ?$

$$\Pi = -mgl \cos \phi \sin \alpha - 3mgl \sin \alpha \cos \phi$$

$$\frac{\partial \Pi}{\partial \phi} = mgl (\sin \alpha - 3 \cos \alpha \sin \alpha) = 0 \Rightarrow \tan \phi = 3 \cot \alpha$$

$$\text{OX: } m: -N_1 \sin \alpha + T \cos(\phi - \alpha) = 0$$

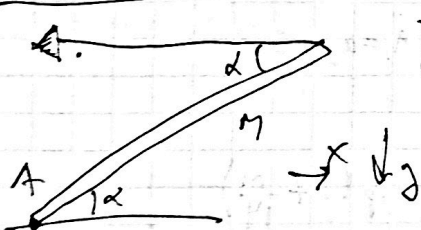
$$\text{OY: } 3m: N_2 \cos \alpha + T \sin(\phi - \alpha) = 0$$

$$-N_1 \cos \alpha + mg + T \sin(\phi - \alpha) = 0$$

$$-N_2 \sin \alpha + 3mg - T \sin(\phi - \alpha) = 0$$

$$N_1 = N_2 \cot \alpha \Rightarrow$$

$T =$

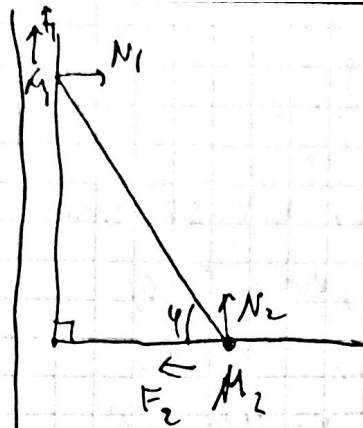


R, mg, T

$$\text{OX: } -T + R_x = 0$$

$$\text{OY: } mg - R_y = 0$$

$$T \sin \alpha = \frac{mg l}{2} \cos \alpha$$



$$\text{OX: } N_1 = F_2$$

$$\text{OY: } mg = N_2 + F_1$$

$$N_1 \cos \alpha + F_1 \cos \alpha = \frac{mg l}{2} \cos \alpha$$

$$mg \leq \mu_1 N_1 = \mu_1 F_2 \leq \mu_1 \mu_2 N_2$$

$$mg \leq N_2 (1 + \mu_1 \mu_2) \quad \text{condition again}$$

New Seminars

Maune Konstante

$$\Pi = \Pi_0 + \frac{\partial \Pi}{\partial q} \bigg|_{q=q_0} (q - q_0) + \frac{1}{2} \frac{\partial^2 \Pi}{\partial q^2} \bigg|_{q=q_0} (q - q_0)^2 + \dots$$

$$\Pi \approx \frac{1}{2} \Pi_1 \eta^2, \quad \eta = q - q_0 \quad \left| \quad k = k - \Pi \right.$$

$$K = \frac{1}{2} k_1 \eta^2, \quad k = k(q, \dot{q}) \quad \left| \quad L \approx \frac{1}{2} (k_1 \dot{\eta}^2 - \Pi_1 \eta^2) \right.$$

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\eta}} \right) - \frac{\partial L}{\partial \eta} = 0 \Rightarrow k_1 \ddot{\eta} + \Pi_1 \eta = 0$$

$$\Rightarrow \omega^2 = \frac{\Pi_1}{k_1}$$

T.e. частота колебаний

new changes:

$$\frac{\partial \Pi}{\partial \eta} \bigg|_0 = 0, \quad \Pi \approx \sum_{i,j} \frac{\partial^2 \Pi}{\partial \eta_i \partial \eta_j} \eta_i \eta_j$$

$$K = \sum \sum \frac{1}{2} k_{ij} \dot{\eta}_i \dot{\eta}_j, \quad \eta_i = \sum_k A_k \cos \omega_k t + \frac{\varphi_k}{\omega_k}$$

$$L = K - \Pi$$

$$\ominus \frac{1}{2} \left\{ \sum_{i,j=1}^s (k_{ij} \dot{\eta}_i \dot{\eta}_j - \Pi_{ij} \eta_i \eta_j) \right\}$$

$$\frac{d}{dt} \left(\sum_{j=1}^s (k_{ij} \dot{\eta}_j + \Pi_{ij} \eta_j) \right) = 0 \quad i=1 \div s. \Rightarrow i=1 \div s$$

$$\text{using change } \eta_i = \sum_k A_k \cos \omega_k t + \frac{\varphi_k}{\omega_k}$$

Решить задачу 1 с помощью:

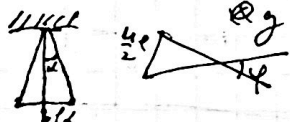
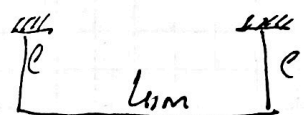
$x(0) = x_0$
 $v(0) = v_0$
 $x_0 = A \cos \varphi$
 $v_0 = -A \omega \sin \varphi \Rightarrow \varphi = -\frac{v_0}{x_0 \omega}$
 и А можно найти с помощью уравнения энергии.

$$\begin{vmatrix} -k_1 x_1^2 + P_{11} & -k_{12} x_1^2 + P_{12} \\ -k_{21} x_2^2 + P_{21} & -k_{22} x_2^2 + P_{22} \end{vmatrix} = 0$$

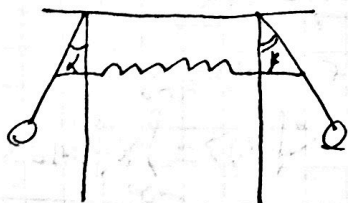
$\{ \omega_k \}$ - собственные.



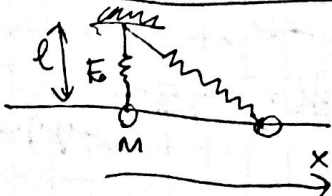
$P = \frac{kx^2}{2} = \frac{x^2}{2} \cdot k$
 $k = \frac{\ddot{x}^2}{2} \cdot m + \frac{\omega^2}{2} \cdot I \stackrel{I = mR^2}{=} \frac{\ddot{x}^2}{2} (k_1)$
 $\ddot{x} = R\ddot{\varphi}$



$P = -mgl \cos \alpha \approx -mgl(1 - \frac{\alpha^2}{2}) = P_0 + mgl \frac{\alpha^2}{2}$
 $k = \frac{m \ddot{\alpha}^2}{2} + \frac{J \ddot{\alpha}^2}{2} \quad \varphi \approx \alpha = \frac{l}{2} \varphi$
 $z \approx l \cos \alpha \approx l - l \frac{\alpha^2}{2} \quad \varphi \sim \alpha \Rightarrow \text{переносим}$
 $v = \dot{z} \approx \dot{\alpha} \sim \alpha \cdot \dot{\alpha} \quad \frac{mv^2}{2}$



Еще две с.з.



$k = \frac{\ddot{x}^2}{2} \cdot m$
 $P = \frac{kx^2}{2}$
 $l^2 + x^2 = (l + \Delta l)^2$

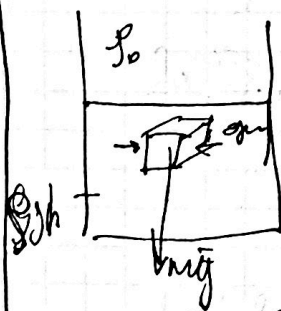
$\alpha = \frac{x^2}{2l}; \quad P_0 = \frac{kx_0^2}{2}, \quad F_0 = kx_0$

$P = \frac{kx^2}{2} = \frac{k}{2} (x_0 + \Delta l)^2 = \frac{k}{2} (x_0^2 + 2x_0 \Delta l + \Delta l^2)$

Рассчитать Δl с помощью

New Seminars

Механика жидкостей и газов.



given - radius $P_1 \neq P_2$
 $P_2 \neq F_2$

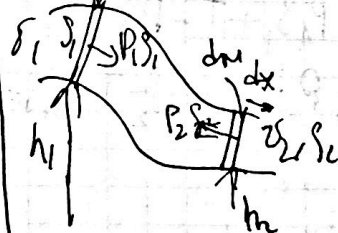
$F_1 + F_2 + mg = 0$

$P_1 S + mg - P_2 S = 0$

$mg = \rho V g = \rho a S g$

$P_2 - P_1 = \rho g h$

$\frac{\rho v^2}{2} + \rho g h + P_0 = \text{const.}$



$\sum A = P_1 S_1 dx_1 - P_2 S_2 dx_2 + dm g (h_1 - h_2)$
 $\Delta E_k = \frac{dm}{2} (v_2^2 - v_1^2)$

$dm = \rho S_1 dx_1 = \rho S_2 dx_2$
 $P_1 - P_2 + \rho g (h_1 - h_2) = \frac{\rho}{2} (v_2^2 - v_1^2)$

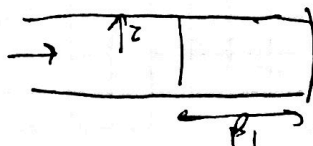
$dV = S_1 dx_1$
 $P_1 - P_2 + \rho g (h_1 - h_2) = \frac{\rho}{2} (v_2^2 - v_1^2)$

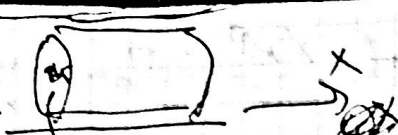
гидравлика:

$V = \frac{\pi R^4 \Delta p}{8 \eta l}$

l - длина трубы

$\Delta p = P_1 - P_2 = \Delta P$
 η - коэффициент вязкости



$$|F| = \eta \frac{dv}{dz} S_{\text{св.м.}}$$


$$\text{Ох: } p_1 S - p_2 S - \eta \frac{dv}{dz} S_1 = 0.$$

$$(p_1 - p_2) \pi r^2 = \eta \frac{dv}{dz} 2\pi r l \quad \text{св.м.}$$

$$-\frac{dp}{dx} = \frac{dv}{dz} \frac{1}{2} \cdot 2\eta$$

$$\text{const} \Rightarrow dv = -\frac{dp}{dx} \cdot \frac{1}{2} 2\eta dz$$

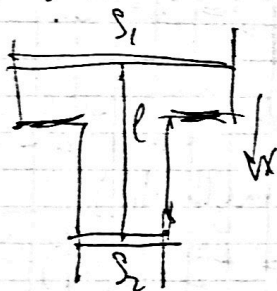
$$\Rightarrow v(z) = -\frac{\Delta p}{2\eta l} \frac{z^2}{2} + C \Rightarrow \frac{\Delta p}{2\eta l} \frac{R^2}{2}$$

$$\Rightarrow v(r) = \frac{\Delta p}{4\eta l} (R^2 - r^2).$$



$$Q = \int dm$$

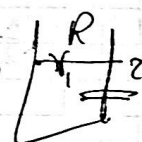
Задача:



Т?

$$\text{Ох: } p_A S_1 + T - p S_1 = 0$$

$$p S_2 + g S_2 l - T - p_A S_2 = 0$$



$$V = \frac{\pi R^4 \epsilon \sigma p}{8\eta l}$$

$$\Delta p = p_A + g S_2 l - \frac{g S_2 l}{2} - p_A + \frac{g S_2 l}{2}$$

$$\Delta p = p_1 - p_2 = p_A + g S_2 l$$

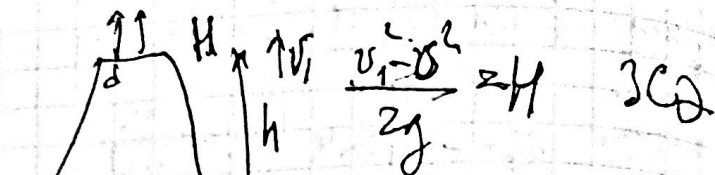
Кин. работа

Состояние. механика

$$dW = w(x) dx \quad \langle f(x) \rangle = \int_{-\infty}^{\infty} x f(x) dx$$

$$\text{Судья: } \langle \exp(-\frac{H}{kT}) \rangle = w,$$

$H(q, \text{уравн. движения})$ $\langle H \rangle$ - средн. значение энергии



$$M = \frac{dm}{dt} = \rho \frac{dv}{dt} = \rho S \frac{dv}{dt}$$

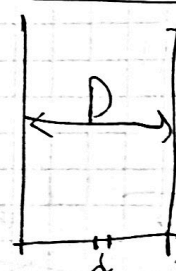
M-?
Δp-?

$$= \rho S_1 v_1$$

$$p + g S_2 h + \frac{\rho v_1^2}{2} = \text{const}$$

$$p_A + g S_2 h + \frac{\rho v_1^2}{2} = p_A + \Delta p + \frac{\rho v_2^2}{2}$$

$$\rho S_1 v_1 = \frac{dm}{dt} = \rho S_2 v_2$$



$$p + g S_2 h + \frac{\rho v_1^2}{2} = p + \frac{\rho v_2^2}{2}$$

Гиббс

Максвелл
 для кан. ансам.
 (при пост. темп. и числе частиц)
 вероятности раз

$$W = C \exp\left(-\frac{m v^2}{2kT}\right)$$

$$W_x = C \exp\left(-\frac{m v_x^2}{2kT}\right)$$

$$P_x(v_x, v_x + dv_x) = W_x dv_x$$

$$C = C_1$$

$$1 = C_1 \int_{-\infty}^{+\infty} \exp\left(-\frac{m v_x^2}{2kT}\right) dv_x \Leftrightarrow \int_{-\infty}^{+\infty} \exp(-x^2) dx = \sqrt{\pi}$$

$$\Leftrightarrow C_1 \int_{-\infty}^{+\infty} \exp\left(-\left(\frac{v_x}{\sqrt{\frac{2kT}{m}}}\right)^2\right) d\left(\frac{v_x}{\sqrt{\frac{2kT}{m}}}\right) = C_1 \sqrt{\frac{2kT}{m}} = 1$$

$$\Rightarrow C_1 = \sqrt{\frac{m}{2\pi kT}} \Rightarrow C = \left(\frac{m}{2\pi kT}\right)^{3/2}$$

$x \in V, N = \infty$ ~~разб.~~ $P, \text{ в } v: n, \text{ на единицу разб.}$

$$\left(\frac{v}{V}\right)^n \cdot \left(\frac{V-v}{V}\right)^{N-n} = P$$

$$C \approx (1/2)$$

$$\Leftrightarrow \frac{N^n}{n!} \left(\frac{v}{V}\right)^n \exp\left(-\frac{vN}{V}\right) \approx \left(\frac{Nv}{V}\right)^n \cdot \frac{1}{n!} \exp\left(-\frac{Nv}{V}\right)$$

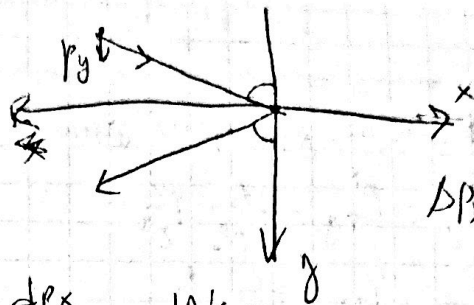
$$\text{Pois}\left(\frac{Nv}{V}\right)$$

$$\alpha = N \cdot v$$

$$\sigma_n = \frac{\sqrt{\sigma_x^2}}{\langle x \rangle} = \frac{\sqrt{x}}{x} = \frac{1}{\sqrt{x}}$$

V, N, T, P ?

Борн



$$\Delta p_x = 2p_x = 2mv_x$$

$$F = \frac{dp_x}{dt}, dN(v_x) = W(v_x) \cdot dv_x$$

$$dV = S \cdot v_x dt, dN = \frac{dV}{v_x} n$$

$$dN(v_x) = dN \cdot W(v_x) dv_x$$

$$dp_x(v_x) = \Delta p_x \cdot dN(v_x) = 2mv_x n dv_x W(v_x) dv_x$$

$$P = \frac{F}{S} = \frac{1}{S} \frac{dp_x}{dt} = \int dv_x (2mv_x^2 W(v_x) n) \Leftrightarrow n = \frac{N}{V}$$

$$\Leftrightarrow m \cdot n \int_{-\infty}^{+\infty} v_x^2 \sqrt{\frac{m}{2\pi kT}} \exp\left(-\frac{m v_x^2}{2kT}\right) dv_x = \langle v_x^2 \rangle$$

$$\Rightarrow \left\langle \frac{m v_x^2}{2} \right\rangle = \frac{kT}{2} \Rightarrow n kT$$

$$u = v_1 - v_2$$

$\langle v_{1x} - v_{2x} \rangle$ сум. 2х скоростей
 тем же темп. разб.

Электричество

$$\textcircled{1} \vec{F}_{12} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r_{12}^2} \vec{e}_{12}$$

Закон Кулона

③ Принцип суперпозиции

④ Основы электромагнетизма: будем рассуждать.

не бегая по полю, а в магн. поле, а не в электр. поле.

Нельзя сказать, что поле магн. является следствием электр. поля.

Эл. поле $\rightarrow$ $\vec{E}$ - хар-на само поле
 $\rightarrow$ $\vec{A}$ - потенциал (или векторный потенциал) - будет с полем

Николай
 Николов
 brant@phystech.ru

Электричество
 и магнетизм
 Космичко, 2006

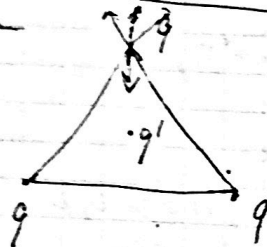
$$\vec{F}_i = \vec{E}_q$$

$$W_q = q \cdot V, \text{ берем ат. зог. члене генов.}$$

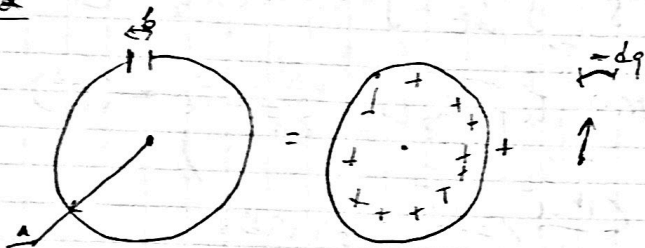
$$\text{Точеч. заряд: } \vec{E}_{T.z.} = \frac{1}{4\pi\epsilon_0} \frac{q\vec{r}}{r^3}$$

$$\epsilon_{T.z.} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$$

$$\Delta_1 \quad \Delta_2 \quad q' \quad 0;$$



Δ_2



$$x = \frac{dq}{dl}, \quad y = \frac{dq}{ds}, \quad z = \frac{dq}{dV}$$

линейная плотность, поверхностная плотность, объемная плотность

$$x = \frac{q}{2\pi a}, \text{ т.к. } q \gg l$$

$$\Rightarrow dq = x dl = \frac{q l}{2\pi a} \Rightarrow k \frac{dq}{a^2} = \epsilon_{T.z.}$$

Δ_3

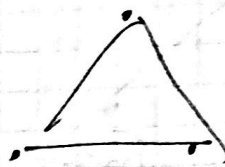
$$\frac{1}{4\pi\epsilon_0} = 9 \cdot 10^9 \text{ н.к.} \Rightarrow \frac{1}{4\pi\epsilon_0} \frac{1}{a} \frac{q}{m} \in$$

$$\ominus \frac{9 \cdot 10^9}{6.6 \cdot 10^{-11}} \left(\frac{1.6 \cdot 10^{-19}}{9.1 \cdot 10^{-31}} \right)^2 \approx 10^{24+24-2} \approx 10^{46}$$

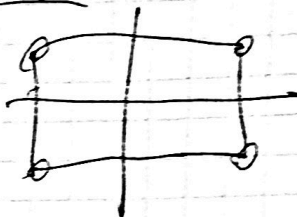
$$ma = F; \Rightarrow a = k \cdot \frac{e^2}{r^2 m} = 9 \cdot 10^9 \frac{2.56 \cdot 10^{-38}}{10^{-6} \cdot 9.1 \cdot 10^{-31}} \approx 2.56 \cdot 10^{13} \text{ м/с}^2$$

$$\ominus 10^8 \sim 10^7 g$$

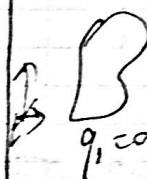
$\Delta 1.7$



$\Delta 1.8$



квадрат



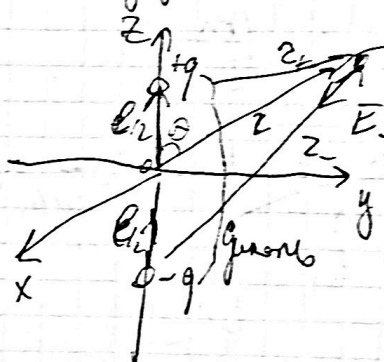
круг

Еще центр. зог.

$$\Rightarrow \vec{p} = \sum q_i \vec{r}_i \text{ - центр. заряд}$$

линейный момент

$\Rightarrow$ квадрат. момент



Вспомогат. ось

$$\vec{E}_+ = \vec{E} - \frac{\vec{E}}{2} \Rightarrow \vec{E}_+ = \frac{q}{4\pi\epsilon_0} \frac{\vec{r}_+}{r_+^3}$$

$$\vec{E}_- = \vec{E} + \frac{\vec{E}}{2} \Rightarrow \vec{E}_- = \frac{-q}{4\pi\epsilon_0} \frac{\vec{r}_-}{r_-^3}$$

$$\vec{E}_+ + \vec{E}_- = \frac{q}{4\pi\epsilon_0} \left(\frac{\vec{r}_+}{r_+^3} - \frac{\vec{r}_-}{r_-^3} \right)$$

$$\frac{1}{(r - \frac{r}{2})^3} \text{ , т.к. } r \gg \frac{r}{2} \Rightarrow \frac{1}{r^3 - \frac{3}{2}r^2 + \frac{3}{4}r - \frac{1}{8}}$$

$$\approx \frac{1}{r^3} \left(1 + \frac{3}{2} \frac{r}{r} + \frac{15}{8} \left(\frac{r}{r} \right)^2 \right)$$

$$\ominus \frac{1}{r^2} \left(1 - \frac{3}{2} \frac{r}{r} + \frac{15}{8} \left(\frac{r}{r} \right)^2 \right)$$

§1.5

$dE = \frac{1}{4\pi\epsilon_0} \cdot \frac{q \, dS}{r^2}$

$\Rightarrow dE_y = dE \cdot \cos \alpha$

$\cos \alpha = \frac{z}{\sqrt{z^2 + b^2}}$

$\Rightarrow E = \int dE_y = \int_0^\infty \frac{1}{4\pi\epsilon_0} \cdot \frac{q \, dz}{(z^2 + b^2)^{3/2}} \cdot \frac{z}{\sqrt{z^2 + b^2}}$

$\Rightarrow F = q \cdot E = \frac{q^2}{4\pi\epsilon_0 b} \cdot l$

§1.10

§1.11

$dq = \sigma \cdot 2\pi a \, d\phi$

$dE_z = \frac{1}{4\pi\epsilon_0} \cdot \frac{\sigma 2\pi a \, d\phi}{z^2 + a^2}$

$E_z = \int dE_z = \int dE \cos \alpha = \int \frac{1}{4\pi\epsilon_0} \cdot \frac{\sigma 2\pi a \, d\phi}{(z^2 + a^2)^{3/2}} \cdot z$

$\Rightarrow \frac{\sigma}{4\pi\epsilon_0} \cdot 2\pi \cdot \frac{1}{z} \int \frac{d(z^2 + a^2) \cdot z}{z^2 (1 + (\frac{a}{z})^2)^{3/2}}$

$\Rightarrow \frac{\sigma}{4\epsilon_0} \int \frac{d(\xi + 1)}{(\xi + 1)^{3/2}} = \left[\frac{-\sigma}{4\epsilon_0} \frac{2}{\sqrt{1 + \frac{a^2}{z^2}}} \right]_{R_1}^{R_2}$

$R_1 = 0, R_2 = \infty \Rightarrow \left[\frac{\sigma}{2\epsilon_0} \right]_{\text{сеч. топ.}} - \text{сеч. дн.}$

$\Rightarrow E = \frac{\sigma}{2\epsilon_0}$

вспомогат.

$\epsilon_{r.3} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r}, W_q = q \cdot \varphi$

$E = -\text{grad} \varphi = -\vec{\nabla} \varphi, \vec{\nabla} = \left\{ \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\}$

$\vec{p} = q\vec{r}, \vec{p}_{\text{сум.}} = Zq_i \vec{r}$

$\varphi = \frac{(\vec{r} \cdot \vec{p})}{r^3} \cdot \frac{1}{4\pi\epsilon_0}$ (или $q=0$)

Т.е. центр. $\Rightarrow$ в центре от центра СК

§2.1

§2.2

$\varphi = \frac{1}{4\pi\epsilon_0} \left(\frac{2q}{a} + \frac{2q}{2a} - \frac{2q}{3a} + \dots \right)$

$\Rightarrow -\frac{q}{2\pi\epsilon_0} \left(1 - \frac{1}{2} + \frac{1}{3} - \dots \right)$

§2.3

$A = W_q|_1^2 = 0 - 0$

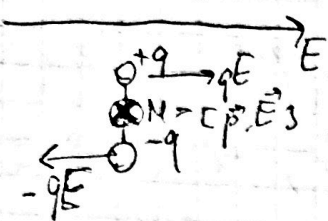
$\Rightarrow$

§2.4

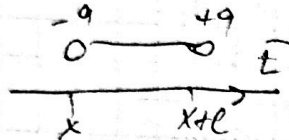
$E = -\frac{\partial \varphi}{\partial x}$

$\varphi = -\frac{\partial \varphi}{\partial x} = \frac{\partial \varphi}{\partial x}$

$$\varphi = -Ex + c$$



Внешнее поле
маленькое. Внешнее.
(4)

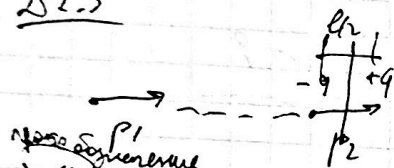


$$W = -q\varphi(x) + q\varphi(x+l)$$

$$\odot q[-E(x+l) + c] + E(x) - c = -pE$$

$$T.e. W = \frac{p \cdot E}{4\pi\epsilon_0}$$

§2.5



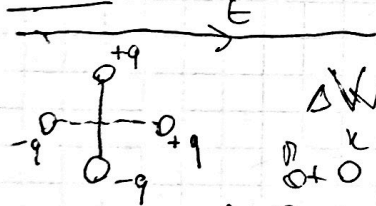
$$W_{P1} = -q \cdot \frac{1}{4\pi\epsilon_0} \frac{1}{r_1} + q \cdot \frac{1}{4\pi\epsilon_0} \frac{1}{r_2} = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{r_2} - \frac{1}{r_1} \right)$$

$$\odot q \left(\frac{1}{(r + \frac{l}{2})^2} - \frac{1}{(r - \frac{l}{2})^2} \right) =$$

$$= \frac{q l}{2} \left(\frac{1}{(1 + \frac{l}{2r})^2} - \frac{1}{(1 - \frac{l}{2r})^2} \right) \approx \frac{q l}{2} \left(1 - \frac{l}{r} - 1 - \frac{l}{r} \right)$$

$$\Rightarrow \frac{p \cdot E}{4\pi\epsilon_0}$$

§2.6



$$\Delta W = 0$$

$$p \cdot E = M \omega^2 l - q l E$$

в равновесии

$$\Rightarrow \omega = \sqrt{\frac{q E}{M l}}$$

$$M \omega = M$$

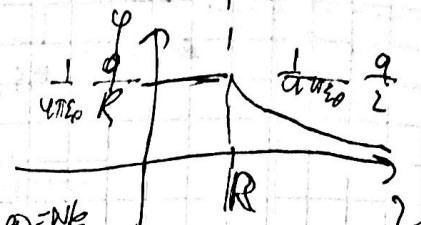
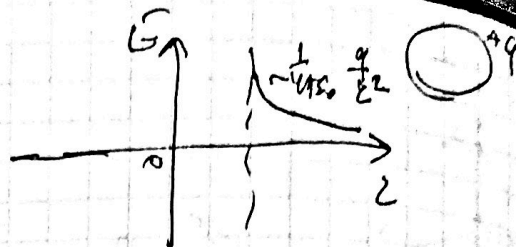
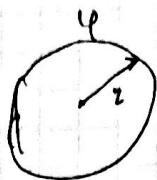
$$\Rightarrow \omega = \frac{q E l}{M l^2}$$

§2.8



$$\oint \frac{1}{4\pi\epsilon_0} \frac{dq}{R} = \frac{1}{4\pi\epsilon_0} \frac{Q}{R}$$

$$\odot \frac{Q}{4\pi\epsilon_0 R}$$



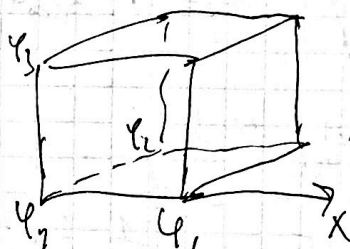
$$\Rightarrow \varphi = \frac{1}{4\pi\epsilon_0} \frac{Q}{R}$$

$$z) N = \frac{Q R}{e} \cdot 4\pi\epsilon_0 \approx 2 \cdot 10^{10} \text{ шт.}$$

$$\text{На ноб.: } E = \frac{Q}{R} \approx 300 \frac{\text{кВ}}{\text{м}} = 3 \frac{\text{кВ}}{\text{см}}$$

Величина
напряж. поля 300 кВ/см

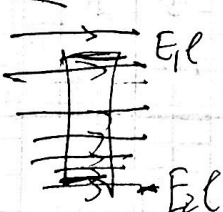
§2.11



$$\vec{E}_4 = \left(\frac{q_1 - q_2}{a}, -1, -1 \right)$$

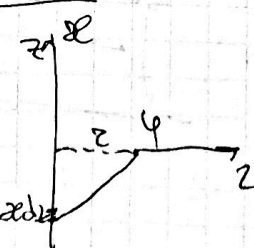
$$E \approx -\frac{\Delta \varphi}{\Delta z}$$

§2.14



$$\Delta A_{\varphi} = (E_1 - E_2) l$$

§2.13



$$d\varphi = \frac{1}{4\pi\epsilon_0} \frac{dq}{R^2}$$

$$\Rightarrow \varphi = \frac{Q}{4\pi\epsilon_0 R}$$

$$\varphi = \frac{2\pi}{4\pi\epsilon_0} \int_0^\infty \frac{dz}{\sqrt{1+z^2}}$$

т.к. на границе цилиндра поле не равно нулю $\Rightarrow$ не работает.

по непрерывности $\Rightarrow$ $\epsilon = \int \vec{E} d\vec{e}$

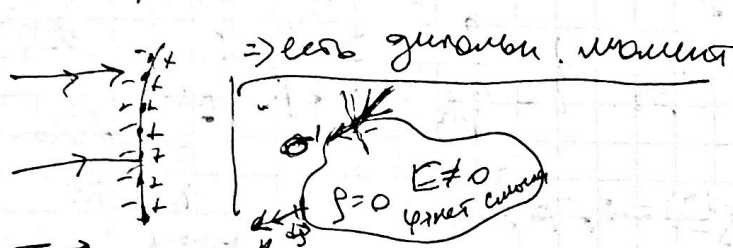
2

кп. работа

Проводник и диэлектрик. Проводник с зарядом q в диэлектрике. Если диэлектрик однородный, то $E=0, \rho=0$ внутри, на поверхности $E \neq 0$.



Диэлектрик. (нет свободн. зарядов)



$$\frac{\sum \vec{p}_i}{\Delta V} \stackrel{\text{def}}{=} \vec{P}, \quad \text{где } \vec{p}_i \text{ — дипольные моменты}$$

Т. Остроградского $\oint \vec{E} d\vec{s} = \frac{q}{\epsilon_0}$ (вектор электрического поля)

$$\vec{D} \stackrel{\text{def}}{=} \epsilon_0 \vec{E} + \vec{P}$$

$$\oint \vec{D} d\vec{s} = q_{\text{своб.}}$$

$$\oint \vec{P} d\vec{s} = -q'$$

Век. упр. $\vec{E}, \vec{D}$: $E_{x1} = E_{x2}$

$$\vec{P} = \epsilon_0 \chi \vec{E}$$

диэлектрик. характеризует

$$\vec{D} = \epsilon_0(1+\chi)\vec{E}$$

характер. проводн. среды

$$\epsilon_1 E_{n1} - \epsilon_2 E_{n2} = \sigma$$

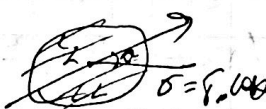
$$\oint \vec{D} d\vec{s} = q_{\text{своб.}} \Rightarrow \text{div } \vec{D} = \rho_{\text{своб.}}$$

$$\text{div } \vec{D} = (\vec{\nabla} \cdot \vec{D}) = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

§ 3.2

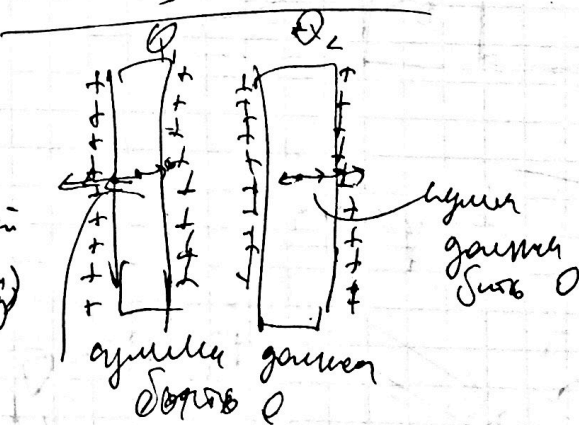
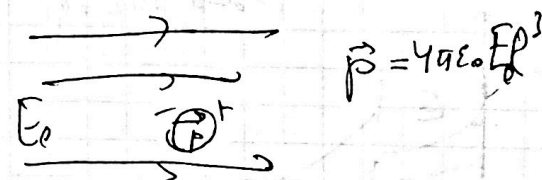
$$\vec{E} = \frac{\sigma}{2\epsilon_0} \quad \vec{E} = \frac{\sigma}{\epsilon_0} \quad \vec{E} = \frac{\sigma}{\epsilon_0}$$

§ 3.2

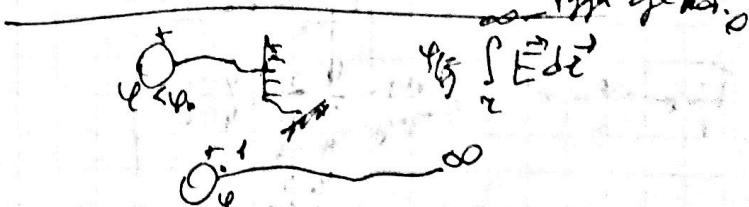
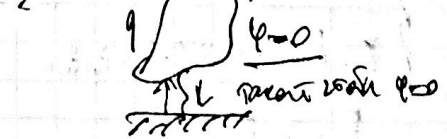


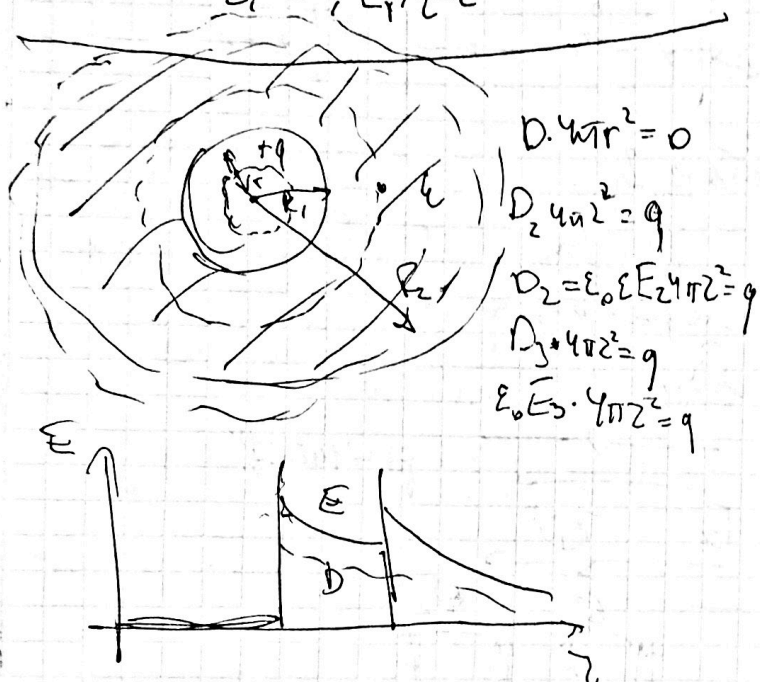
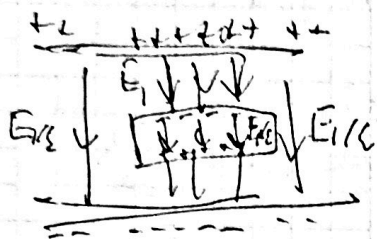
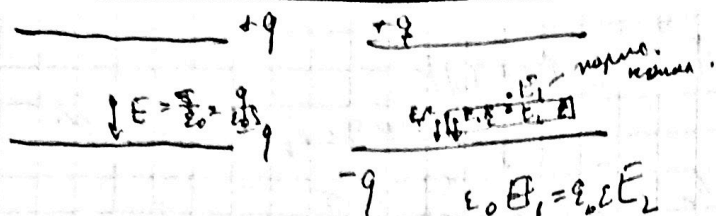
$$E = \frac{\sigma}{3\epsilon_0}$$

§ 3.2

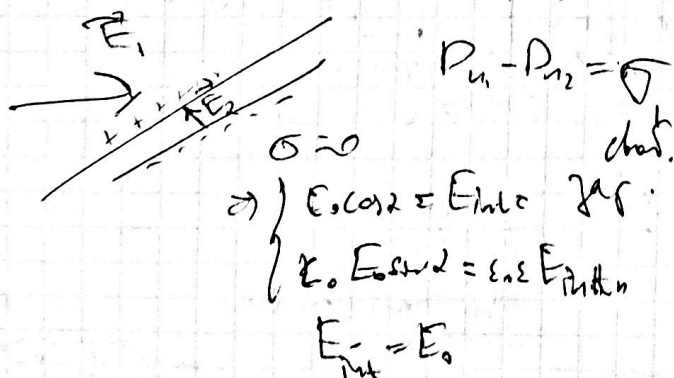


§ 3.2





$\sqrt{3.20}$
 кр. паролл



Грани. Д1
 4.2 4.21
 4.10 4.22
 4.15
 4.20

Грани. Д2
 4.3, 4.5, 4.6, 4.11, 4.15, 4.16

Грани. Д3
 4.7, 4.8, 4.9, 4.12, 4.13, 4.14, 4.23

$\Delta 4.18$
 $\nabla^2 \varphi = -\frac{\rho}{\epsilon_0 \epsilon}$
 $\nabla^2 \varphi = -\frac{\rho}{\epsilon_0 \epsilon}$
 не модно



$\Delta 4.19$
 $\varphi = \begin{cases} \frac{q}{4\pi\epsilon_0 R}, & z \geq R \\ -\frac{qz^2}{2\epsilon_0 R^3} + \frac{q}{2\epsilon_0 R}, & z < R \end{cases}$

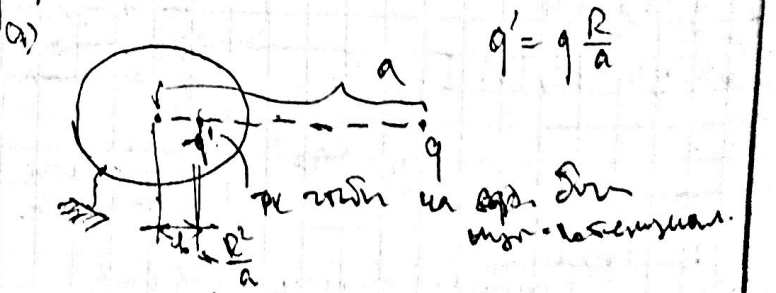
$\frac{1}{z^2} \frac{\partial}{\partial z} (z^2 \frac{\partial \varphi}{\partial z})$
 $\frac{\partial \varphi}{\partial z} = \begin{cases} -\frac{q}{2\epsilon_0 R^3}, & z \geq R \\ -\frac{qz}{\epsilon_0 R^3}, & z < R \end{cases}$

$\varphi(z) = -\frac{q}{2\epsilon_0 R^3} \frac{z^2}{2}$
 $\varphi(z) = -\frac{q}{2\epsilon_0 R^3} \frac{z^2}{2} \frac{\partial \varphi}{\partial z}$

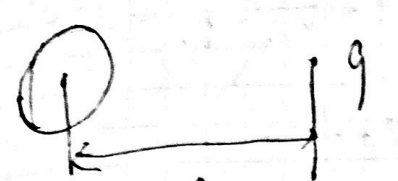
$\varphi = \begin{cases} 0, & z \geq R \\ \frac{q}{2\epsilon_0 R^3} \frac{3az^2}{R^3}, & z < R \end{cases}$

$\varphi = \frac{3q\epsilon_0}{R^3}$

4.4



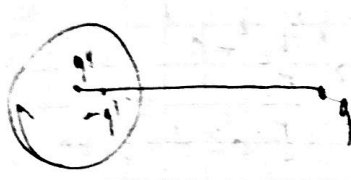
и сфер. не заряжен. и и шар.



$$q' = -q \frac{R}{a},$$

$$q'' = Q + q \frac{R}{a}$$

$\frac{1}{4\pi\epsilon_0} \frac{q}{a}$ и сфер. нос. с. к.
 кол. в. $\varphi = 0$
 осев. q



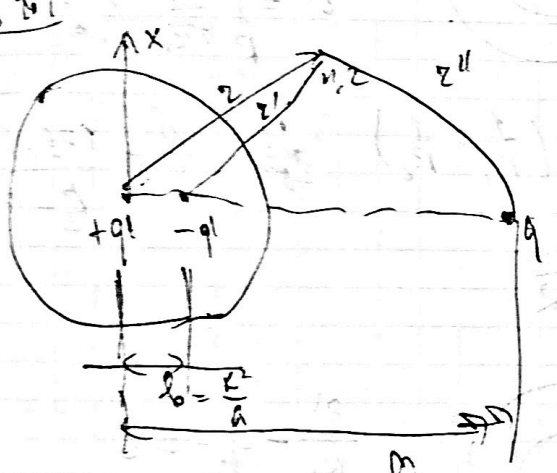
$$\frac{1}{4\pi\epsilon_0} \frac{q'}{R} = \frac{1}{4\pi\epsilon_0} \frac{q}{a}$$

$$\Rightarrow q' = q \frac{R}{a}$$



$$\frac{Q}{R} + \frac{q}{a} = \frac{Q''}{R}$$

54.4



$$\sigma = \epsilon_0 \epsilon E_n|_{z=R}, \quad \sigma = \epsilon_0 E_z|_{z=R} = 0 =$$

$$D_{nz} = \epsilon_0 \frac{\partial \varphi}{\partial z}|_{z=R}$$

$$\frac{1}{4\pi\epsilon_0} \varphi(z) = \frac{q'}{z} - \frac{q'}{z'} + \frac{q''}{z''}$$

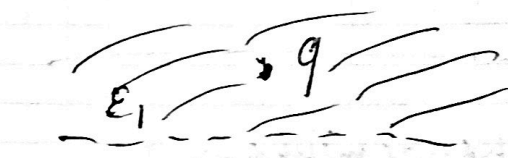
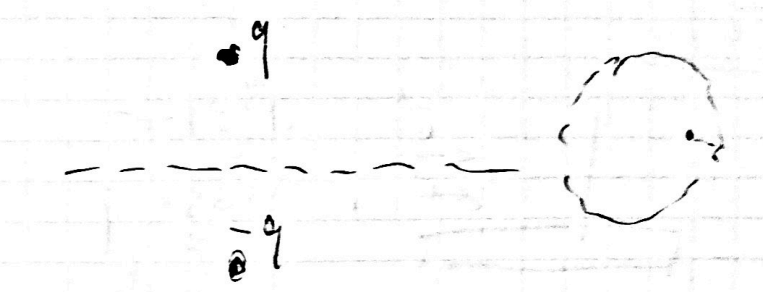
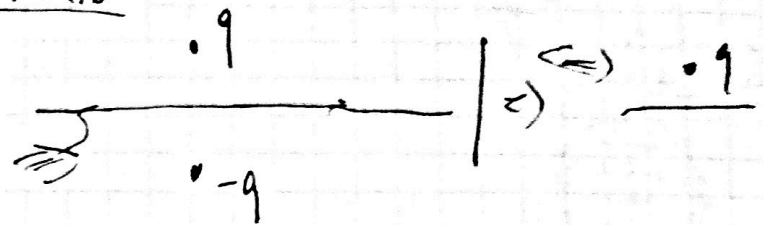
$$z' = \sqrt{z^2 + b^2 - 2zb \cos \theta}, \quad z'' = \sqrt{z^2 + a^2 - 2za \cos \theta}$$

$$\frac{\partial}{\partial z} \left(\frac{1}{z} \right) = -\frac{1}{z^2}$$

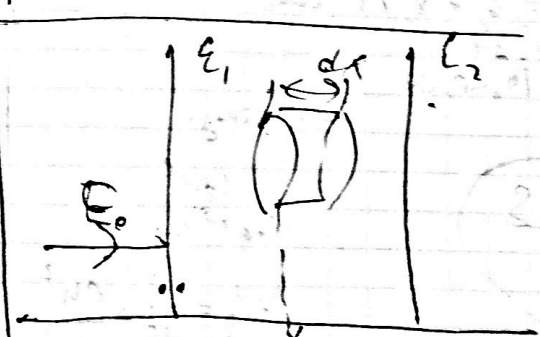
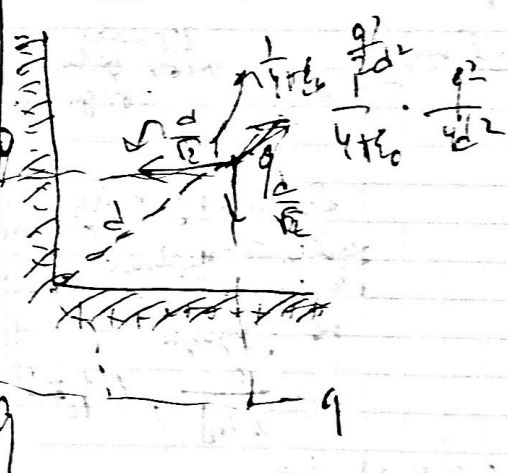
$$\sigma = -\epsilon_0 \frac{1}{4\pi\epsilon_0} \left(-\frac{q'}{z^2} + \frac{q'}{z'^2} - \frac{q''}{z''^2} \right) \Big|_{z=R}$$

$$q' = q \frac{R}{a}$$

54.6



54.6



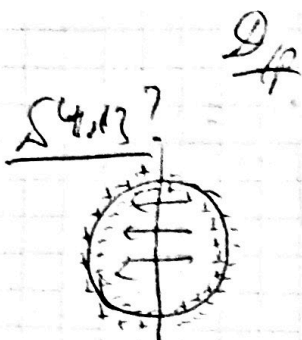
$$\epsilon \cdot E = \epsilon_0 \epsilon(x) \cdot E(x)$$

$$\frac{dE}{dx} = \frac{q'(x)}{\epsilon_0}$$

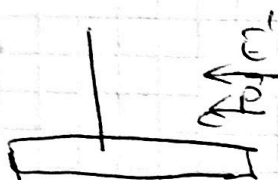
$$\oint \vec{D} d\vec{S} = q - q' = 0$$

$$\oint \vec{E} d\vec{S} = \frac{q}{\epsilon_0} - \frac{q'}{\epsilon_0}$$

$$\epsilon(x) = \epsilon_1 + \frac{x}{\epsilon_2 - \epsilon_1}$$



54.14



$$(\vec{P}, \vec{n}) = \sigma$$

$$\oint \vec{P} d\vec{s} = -q'$$

$$\vec{P} = \vec{E} \epsilon_0$$

$\epsilon^2 \frac{\partial \varphi}{\partial z}$ - напряженность.

Кин. работа

Эл. поле

усл. проб.
Зел. в устан
до-б. проб.
 $C_{11} = \frac{q}{U}$

плоск.
меш. поле
от грав.

Конг.

одн. конг. - грав.
зрелость - перем. в грав.
и грав. обр. грав.
на грав. грав.

$$\hat{e}_{12} = \frac{q}{4\pi\epsilon_0 r_{12}}, \Delta\varphi_2 = U$$

- | | | |
|---------------------|----------------------|------|
| $\Gamma_p \sqrt{1}$ | 1, 5, 7, 6, 7, 8, 23 | эле. |
| $\Gamma_p \sqrt{2}$ | 15, 17, 21 | |
| $\Gamma_p \sqrt{3}$ | 2, 18, 19, 20, 22 | |
| $\Gamma_p \sqrt{4}$ | 9-12 | |

Стор. конг.

$$|E| C = \frac{\epsilon_0 \epsilon S}{C}$$

+q -q

$$W_g = \frac{q^2}{2C} = \frac{CU^2}{2}$$

$$\frac{\epsilon_0 \epsilon E^2}{2} = W_E = \frac{dW}{dV} - \text{объемная плотность энергии}$$

Почему, согласно: на конг. один и тот же заряд - т.н.т. - $\frac{1}{C_{об}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$
 Парал. : одно и то же напрел. $\frac{1}{C_{об}} = C_1 C_2 + \dots$



$$\varphi = \int \vec{E} d\vec{r}$$

$$\varphi = \frac{1}{4\pi\epsilon_0} \left(\frac{q_1}{r_1} + \frac{q_2}{r_2} \right) \Rightarrow \int \frac{q dv}{r}$$

$$\oint \varphi = \frac{q}{\epsilon_0}$$



$$\epsilon_0 E_1 \cdot 4\pi r^2 = q - \text{не имеет смысла}$$

$$\epsilon_0 E_2 \cdot 4\pi r^2 = q - \text{смысл имеет}$$

$$\varphi(R) = \int_{R_1}^R E_1 dz + \int_{R_2}^{\infty} E_2 dz \ominus$$

$$\ominus \frac{q}{4\pi\epsilon_0} \left(\frac{1}{\epsilon z} \right)_{R_1}^{R_2} + \frac{1}{2} \left(\frac{R_1}{R_2} \right) \Rightarrow \text{инкаст}$$

Эквив. заряд



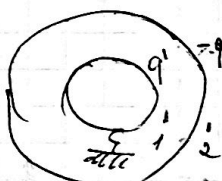
$$q = CU$$

$$U = \int_{R_1}^{R_2} \frac{q}{4\pi\epsilon_0} \cdot \frac{1}{r^2} dz =$$

$$= \frac{q}{4\pi\epsilon_0} \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\frac{q}{4} = \frac{4\pi\epsilon_0 R_1 R_2}{R_2 - R_1}$$

$$W_g = \frac{CU^2}{2}$$



$$\frac{q_1}{R_1} - \frac{q_2}{R_2} = 0$$

$$E_1 = \frac{1}{4\pi\epsilon_0} \frac{q_1}{r^2}, E_2 = \frac{1}{4\pi\epsilon_0} \frac{q_1 - q_2}{r^2}$$

$$W = \int \frac{\epsilon_0 E^2}{2} dV = \int_{R_1}^{R_2} \frac{\epsilon_0 E_1^2}{2} 4\pi r^2 dz + \int_{R_2}^{\infty} \frac{\epsilon_0 E_2^2}{2} 4\pi r^2 dz$$

не зависит
 $\frac{1}{C_{об}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$
 парал. : одно и то же напрел. $\frac{1}{C_{об}} = C_1 C_2 + \dots$



$$\Delta A \Delta W = A_{\text{внеш.чл}} \ominus$$

$$\ominus \Delta A^{\text{ext}} + \Delta A^{\text{int, д.с}}$$

д.ч.ч. $\frac{q^2}{2C}$ $\frac{q^2}{2C}$

для q-го

$$\left(\frac{1}{2} \epsilon \epsilon_0 U^2 - \frac{\epsilon \epsilon_0 U^2}{2} \right) = \Delta A + U \epsilon \epsilon_0 U^2$$

$$-\frac{1}{2}(\epsilon-1)\epsilon U^2 = \Delta A - (\epsilon-1)\epsilon U^2$$

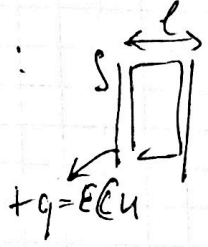
ΔW

$$\Delta A_{\text{в.ч.}} = -2\Delta W_0$$

$$\Delta A = \Delta W_0$$

2) $\epsilon_0, \epsilon, U, q$

2) $\epsilon_0, \epsilon, U, q$

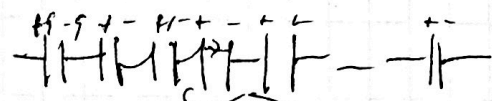


$$C = \frac{\epsilon_0 s}{l}$$

$$\Delta A = \left(\frac{q^2}{2C} - \frac{q^2}{2\epsilon C} \right)$$

$$= \frac{\epsilon^2 C^2 U^2}{2C} \cdot \frac{\epsilon-1}{\epsilon} = \frac{CU^2}{2} \epsilon(\epsilon-1)$$

и $\Delta W_0 = \frac{CU^2}{2}$



$$U = n \frac{q}{\epsilon}$$

$$U' = (n-1) \frac{q}{\epsilon}$$

2/

εε.E

6.9

$$R = \frac{U}{I}, \quad U = \varphi_1 - \varphi_2 = \frac{q_1}{C_1} - \frac{q_2}{C_2}; \quad I = \oint \vec{j} dS = \int \frac{\lambda dS}{\epsilon \epsilon_0} = \frac{\lambda}{\epsilon \epsilon_0} q_1$$

конусная поверхность
с радиусом r

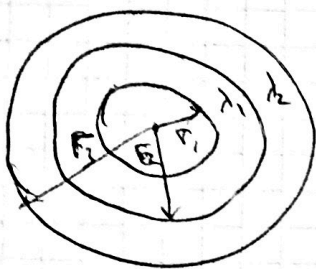
λE — это заряд внутри конуса:

$$I = \frac{\lambda}{\epsilon \epsilon_0} q_2$$

Т.е. ток равен и направлением
в opposite direction $\Rightarrow q_1 = -q_2$

$$\Rightarrow U = q_1 \left(\frac{1}{C_1} + \frac{1}{C_2} \right) \Rightarrow R = \frac{\epsilon \epsilon_0}{\lambda} \left(\frac{1}{C_1} + \frac{1}{C_2} \right)$$

6.10



$$P = I^2 R, \quad R = \frac{U}{I}, \quad U' = (\varphi_1 - \varphi_2) + (\varphi_2 - \varphi_3) =$$

$$= I(R_1 + R_2); \quad E = \begin{cases} \frac{q}{4\pi\epsilon_0\epsilon_1 r^2} & r_1 < r < r_2 \\ \frac{q}{4\pi\epsilon_0\epsilon_2 r^2} & r_2 < r < r_3 \end{cases}$$

$$I = \frac{\lambda l}{\epsilon \epsilon_0} q = \frac{\lambda l}{\epsilon_2 \epsilon_0} q$$

$$\Rightarrow U_1 = \int_{r_1}^{r_2} E dr = \frac{q}{4\pi\epsilon_0\epsilon_1} \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

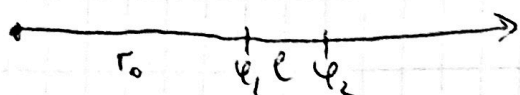
$$U_2 = \int_{r_2}^{r_3} E dr = \frac{q}{4\pi\epsilon_0\epsilon_2} \left(\frac{1}{r_2} - \frac{1}{r_3} \right)$$

$$\Rightarrow R_1 = \frac{U_1}{I}, \quad R_2 = \frac{U_2}{I}; \quad R_1 = \frac{1}{4\pi\lambda_1} \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

$$R_2 = \frac{1}{4\pi\lambda_2} \left(\frac{1}{r_2} - \frac{1}{r_3} \right)$$

$$\Rightarrow P_1 = I^2 R_1 = \frac{U^2 R_1}{(R_1 + R_2)^2}, \quad P_2 = \frac{U^2 R_2}{(R_1 + R_2)^2}$$

6.11



$$U = \varphi_1 - \varphi_2 = \int_{r_0}^l E(r) dr;$$

$$I = \oint \vec{j} dS = \lambda E \cdot \frac{2\pi r l}{2}$$

$$\oint D dS = q \Rightarrow \epsilon \epsilon_0 E \cdot 2\pi r l = q \Rightarrow E = \frac{q}{2\pi \epsilon \epsilon_0 r l}$$

$$\Rightarrow I = \frac{\lambda q}{2\pi \epsilon \epsilon_0 r l} \cdot \frac{2\pi r l}{2} = \frac{\lambda q}{2\epsilon \epsilon_0} \Rightarrow \rho = \frac{2\pi \epsilon \epsilon_0}{\lambda l} \Rightarrow E = \frac{1}{\lambda l \epsilon \epsilon_0} \cdot \frac{\lambda q}{2} =$$

$$\Rightarrow U = \int_{r_0}^l \frac{I}{2\pi \lambda l r} dr = \frac{I}{2\pi \lambda l} \ln \left(\frac{r_0 + l}{r_0} \right) \Rightarrow = \frac{I}{2\pi \lambda l} \ln \left(\frac{r_0 + l}{r_0} \right)$$

$$\lambda = 2 \cdot 10^{-7} \frac{1}{\text{cm} \cdot \text{cm}} = 2 \cdot 10^{-2} \frac{1}{\text{cm} \cdot \text{m}} \quad \approx 14,9 \text{ B.}$$

$$\begin{aligned} r_0 &= 1 \text{ m}, \quad l = 0,75 \text{ m}, \\ U &= 300 \text{ V}, \quad I = 500 \text{ A} \end{aligned}$$

Σ 6.11

$$R = 2 R_{\text{ind}} + R_{\text{ext}}, \text{ με } 3.6.5: R_{\text{ext}} = \frac{\epsilon \epsilon_0}{\lambda C_{\text{ext}}}, \text{ με}$$

$$C_{\text{ext}} = \frac{l}{\lambda \pi D \delta}; \quad C_{\text{ext}} = \frac{\epsilon \epsilon_0 \int}{\delta} = \frac{\epsilon \epsilon_0 \pi D \delta}{l} \quad (2)$$

$$R_{\text{ind}} = \frac{U}{I}, \quad U = \int_{d/2}^{D/2} E dr, \quad \text{Από Γ. Γαργανά: } E = \frac{\rho}{2\pi \epsilon \epsilon_0 r}$$

$$I = \int j ds = \frac{\rho \lambda}{2\pi \epsilon \epsilon_0 r} \cdot 2\pi r \delta = \frac{\rho \delta}{\epsilon \epsilon_0} \Rightarrow E = \frac{I}{2\pi \epsilon \epsilon_0 \delta} \cdot \frac{\epsilon \epsilon_0}{\lambda \delta} = \frac{I}{2\pi \lambda \delta r}$$

$$\Rightarrow U = \frac{I}{2\pi \lambda \delta} \ln \frac{D}{d} \Rightarrow R_{\text{ext}} = \frac{1}{2\pi \lambda \delta} \ln \frac{D}{d} \quad \Rightarrow R = \frac{2}{2\pi \lambda \delta} \ln \frac{D}{d} + \frac{l}{\lambda \pi D \delta} = \frac{1}{\pi \lambda \delta} \left(\frac{l}{D} + \ln \frac{D}{d} \right).$$

Σ 6.12

$$W = \lambda E^2 = \text{const}, \quad I = \int j ds = j(x) 4\pi x^2 = \text{const}$$

$$E(x) = \frac{j(x)}{\lambda(x)} = \frac{I}{4\pi x^2 \lambda(x)} \Rightarrow W = \frac{I^2}{16\pi^2 x^4 \lambda(x)} = \text{const} \Rightarrow \lambda(x) = \frac{k}{x^4}$$

Σ 6.21

$$P = \frac{U^2}{R} = U^2 \cdot \frac{\lambda C}{\epsilon \epsilon_0} = \frac{U^2 \lambda}{\epsilon \epsilon_0} \cdot 4\pi \epsilon \epsilon_0 \frac{R_1 R_2}{R_2 - R_1} = \frac{4\pi U^2 \lambda R_1 R_2}{R_2 - R_1}$$

Σ 6.22

$$C \text{ αγωγιμότητα } I = -\frac{dq}{dt}, \text{ με γρηγορά: } I = \frac{U}{R} = \frac{q}{RC} = \frac{\lambda q}{\epsilon \epsilon_0}$$

$$\Rightarrow \frac{dq}{dt} + \frac{\lambda q}{\epsilon \epsilon_0} = 0 \Rightarrow \frac{dq}{q} = -\frac{\lambda}{\epsilon \epsilon_0} dt \quad q(t) = q_0 e^{-\frac{\lambda}{\epsilon \epsilon_0} t}, \quad q_0 = U_0 C$$

$$\Rightarrow I(t) = \frac{\lambda q(t)}{\epsilon \epsilon_0} = \frac{\lambda U_0 C}{\epsilon \epsilon_0} e^{-\frac{\lambda}{\epsilon \epsilon_0} t}, \quad C = \frac{2\pi \epsilon \epsilon_0 l}{\ln R_2 / R_1} \Rightarrow I(t) = \frac{2\pi \lambda l U_0}{\ln R_2 / R_1} \quad (2)$$

$$\odot e^{-\frac{\lambda}{\epsilon \epsilon_0} t}$$

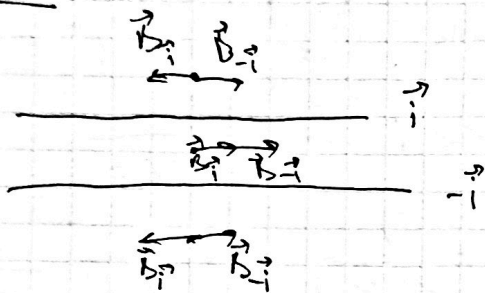
§ 6.23

$$c(t) = \frac{\epsilon_0 S}{v(t+d_0)}, \quad q(t) = \epsilon c(t), \quad I_A = \frac{dq}{dt} = \epsilon \epsilon_0 S \left(\frac{1}{v(t+d_0)} \right)' = -\frac{v \epsilon \epsilon_0 S}{(v(t+d_0))^2}$$

$$\Delta q = c_2 \epsilon - c_1 \epsilon = \epsilon \epsilon_0 S \left(\frac{1}{v_1+d_0} - \frac{1}{v_0} \right) < 0, \quad I = -\frac{dq}{dt} = -I_A$$

Д/р

§ 7.3



Из зак. С2: $B = \frac{\mu_0 I}{2}$

$\Rightarrow$ выходы магнит: $\frac{\mu_0 I}{2} + \frac{\mu_0 I}{2} = \mu_0 I$

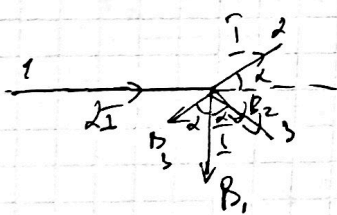
вне: 0

Направлен-е индукц-ии с нормалью
зак. С2 (взят или Ox или $-Ox$
по правилу).

§ 7.5

Задача не изменится, если её заменить на следующую:
по поверх. провода (или радиусу) течёт ток $I + I_0$, а по
поверхности радиуса $-I_0$. Тогда ток в цепи будет равен $\frac{I}{2\pi R} \cdot d$ и
 B соответ-но равно $\frac{\mu_0}{2\pi} \cdot \frac{I}{2\pi R} d = \frac{\mu_0 I d}{4\pi^2 R}$.

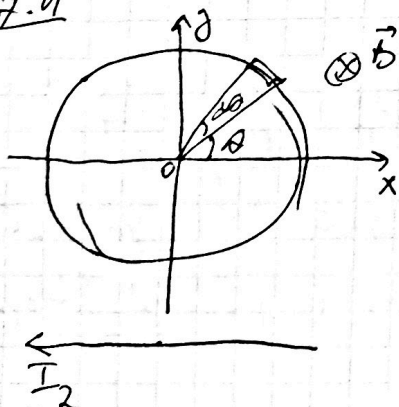
§ 7.9



Из примера 7.1: $B_1 = \frac{\mu_0 I}{2\pi a}$ и $B_2 = B_3 = \frac{1}{2} B_1$

Из зак. С2 $B \perp I$ (на рисунке) $B = B_1 + 2B_2 \cos \alpha =$
 $= \frac{\mu_0 I}{2\pi a} (1 + \cos \alpha)$

§ 7.11



Из примера § 7.1: $B = \frac{\mu_0 I_2}{2\pi(a + R \sin \theta)}$

$dF = I_1 I_2 d\vec{l}_1 d\vec{l}_2$

$F_B = I_1 \int_0^{2\pi} \frac{\mu_0 I_2 R \sin \theta}{2\pi(a + R \sin \theta)} da = \frac{I_1 \mu_0 I_2 R}{2\pi} \int_0^{2\pi} \frac{1}{a + R \sin \theta} d\theta$

$= \frac{\mu_0 I_1 I_2 R}{2\pi} \left(2\pi - a \int_0^{2\pi} \frac{1}{a + R \sin \theta} d\theta \right) = \dots$

$$F_x = \frac{\mu_0 I_1 I_2}{2a} \int_0^{2\pi} \frac{R \cos \alpha}{a + R \sin \alpha} d\alpha = \frac{\mu_0 I_1 I_2}{2a} \ln(a + R \sin \alpha) \Big|_0^{2\pi} = 0.$$

$F_y = F_y$.

§ 7.12

Частицы будут двигаться по окружности.

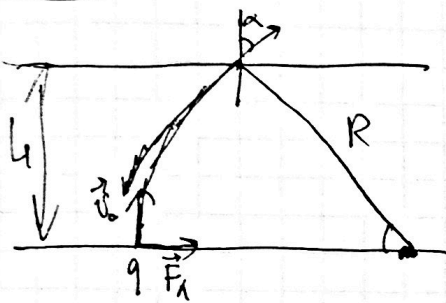
$$\vec{F} = qE\vec{r}, \vec{B}\vec{r} = qv \sin \alpha \vec{r} = m\vec{a}, a = \frac{v_{\perp}^2}{R} = \frac{v^2 \sin^2 \alpha}{R} \Rightarrow R = \frac{m v \sin \alpha}{Bq}$$

$$\Rightarrow T = \frac{2\pi R}{v_{\perp}} \Rightarrow h = v_{\perp} T = \frac{2\pi m v \sin \alpha}{Bq \sin \alpha} \cos \alpha = \frac{2\pi m v \cos \alpha}{Bq}$$

Первый раз встретятся на расстоянии h от т. Встретятся через $T = \frac{2\pi m v \sin \alpha}{Bq \sin \alpha} = \frac{2\pi m}{Bq}$. Макс. расстоянием в

процессе движения будет $4R$ (рис.)

§ 7.13



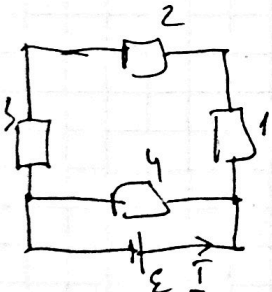
$$m \frac{v_0^2}{R} = qv_0 B \Rightarrow R = \frac{mv_0}{qB}$$

Для случая: 1) $\alpha = \pi$, т.е. $R \leq h$ и т.д.

$$v_0 \leq \frac{qBh}{m}$$

$$2) K > h \Rightarrow \sin \alpha = \frac{h}{R} = \frac{h q B}{m v_0}$$

§ 7.15

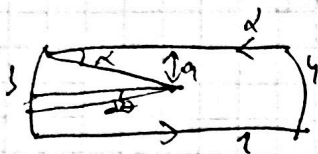


$$E = I_4 R = I_1 (3R) \Rightarrow I_4 = \frac{3I}{4}, I_1 = \frac{I}{4}$$

$$\text{По правилу § 7.1: } B = \frac{\mu_0 I_0}{4\pi \frac{a}{2}} \int_{\frac{a}{4}}^{3\frac{a}{4}} \sin \alpha d\alpha = \frac{\mu_0 I_0}{4\pi a} \cos \frac{\pi}{4} \Rightarrow B_{\text{эквив.}} = -B_4 + B_1 + B_2 + B_3 =$$

$$= -\frac{1}{4} \frac{\mu_0 I}{\pi a} \cos \frac{\pi}{4} + 3 \cdot \frac{\mu_0 I}{4\pi a} \cos \frac{\pi}{4} = 0.$$

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$$B_{12} = \frac{\mu_0 I}{2\pi a} \sqrt{1 - \frac{a^2}{R^2}} \quad \text{wg 7.1}$$

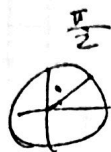
$$B_{324} = \frac{\mu_0 I}{4\pi R^3} \oint_0^{2\pi} R^2 d\phi = \frac{\mu_0 I}{4\pi R} 2\pi$$

$$\Rightarrow B_{\text{gesamt}} = \sum_i B_i = 2 \frac{\mu_0 I}{2\pi a} \sqrt{1 - \frac{a^2}{R^2}} + \frac{\mu_0 I}{\pi R}$$

$$m\vec{a}' = \vec{F} + \vec{F}_{\text{Cor.}}$$

prüfen.

$$\vec{F}_{\text{Cor.}} = m(\vec{a}' - \vec{a}) = m(-\vec{a}_0 - 2\vec{\omega} \times \vec{v}' + \omega^2 \vec{r}'_{\perp})$$



sufficiently well) Interpreter - ges. regel
well enough

$$\frac{\partial}{\partial \vec{r}} \left[\frac{(\vec{\Sigma} \vec{a})}{\vec{r}} \right] = \frac{\partial}{\partial \vec{r}} \left[\frac{\vec{\Sigma} \vec{a}}{\vec{r}} \right] + \frac{\partial}{\partial \vec{r}} \left[\frac{\vec{r} \vec{a}}{\vec{r}^3} \right] = \frac{1}{\vec{r}} \vec{a} - \frac{3\vec{r} \vec{a}}{\vec{r}^4} \cdot \frac{\vec{r}}{2}$$

mit Koordinaten.

→

///

any

my

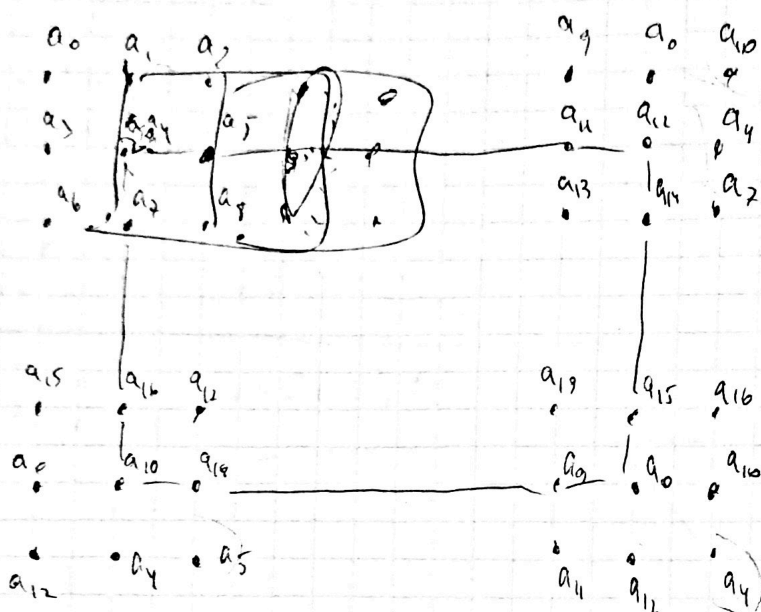
$$y_k = \sum_{i=1}^n \lambda_{ki} x_i$$

$$L = (L_1, \dots, L_k)$$

$$L(-1, 2, -1)$$

$$y_k = \Delta x_{k+1} - \Delta x_k$$

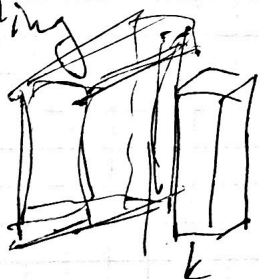
аргументов



U-NET.

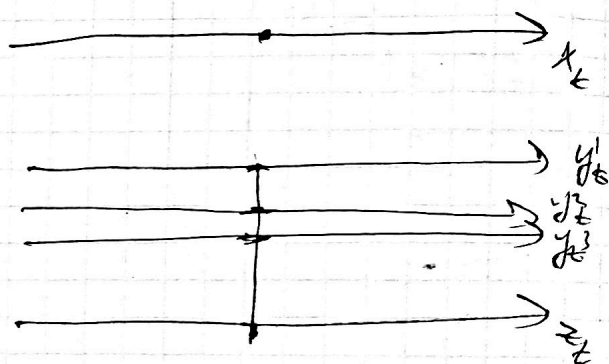
свёртки по пространству $\Rightarrow$ pooling images

Global pooling

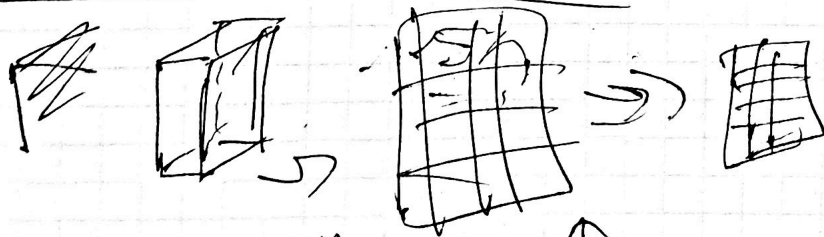


по i in k
составим
сумму.

$\Rightarrow$ - возвращает. код.



2 слоя $\Rightarrow$ отменяет
свёртку



сделаем
на квант

настраиваем сеть по
разм. $\Rightarrow$ стандарт. выход

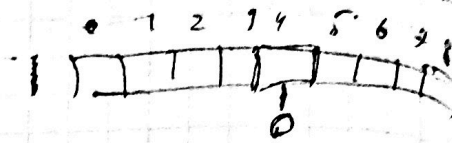
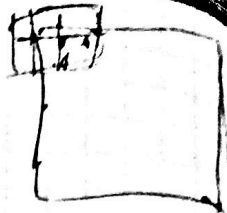
def generate_m1():

matrix.append(
return m

matrix.append(
return matrix

def gener-m0():

~~def~~ ~~gener~~



0 1 2 3 4 5 6 7 8

for i1:
for i2:
for i3:



for i1:
for i2:

Зачеты Крашевских Киритов Нейронные сети.



Смотрим на признаки через более глубокое ман.
реконструирование

1) .

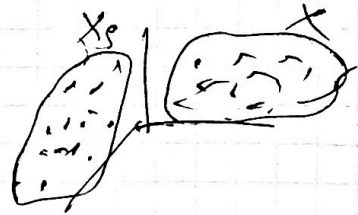
2)

$$x^* = \underset{x \in \mathbb{R}^{n_{\text{input}}}}{\text{argmin}} \| \Phi(x) - \Phi_0 \|_2^2 / \| \Phi_0 \|_2^2 + R_1(x) + R_2(x)$$

$$\mathcal{L}_S(x) = \left\| \sum_{i=1}^n (x_i - \mu_i) \right\|_2^2$$

$$\sum_{i=1}^n = \mathbb{E} (x_i - \mu_i) (x_j - \mu_j)$$

$$\sum_{i,j} = \mathbb{E} x_i x_j$$



Матрица ковариации

Johnson :

$$x \mapsto \boxed{x} \rightarrow f_w(x) = x \quad \text{loss: } L_c(x_c, x) + 2L_S(x_s, x)$$

... нормализуем. $\cap = L + \dots$

$$L = \frac{\Phi}{I}, \quad L_{12} = \frac{\Phi_{21}}{I_1}; \quad W_m = \frac{LI^2}{2} = \frac{I\Phi}{2}$$

2 контура!

$$W_m = \frac{1}{2} (I_1 \Phi_1 + I_2 \Phi_2) = \frac{L_{11} I_1^2}{2} + L_{12} I_1 I_2 + \frac{L_{22} I_2^2}{2}$$

$$\Phi = \int B_n dS$$

объем. пл

$$W_m = \frac{B^2}{2\mu_0} = \frac{1}{2} B H = \frac{1}{2} \mu_0 H^2$$

Работа по перем. пл. с током:

$$\delta A = \int I d\vec{\ell}, \vec{B} \quad \delta \vec{\ell} = \int I d\vec{\ell}, d\vec{\ell} \int \vec{B} = \int I \vec{B} d\vec{\ell} = I \delta \Phi$$

C, L_1
R сопротивление
контур

$$\mathcal{E}_i = \frac{d\Phi}{dt}$$

$$\mathcal{E}_s = \frac{d\Phi}{dt} = L \frac{dI}{dt} = L \dot{I}$$

$$\text{rot } \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

$$\delta A = \int I \mathcal{E}_i dt = - I \delta \Phi, \quad \Delta Q = \frac{\Delta \Phi}{R}$$

$$\begin{cases} \text{div } \vec{B} = 0 \\ \text{div } \vec{D} = \rho \\ \text{rot } \vec{H} = \vec{j} + \frac{\partial \vec{D}}{\partial t} \\ \text{rot } \vec{E} = - \frac{\partial \vec{B}}{\partial t} \end{cases}$$

$$\begin{cases} \int \vec{B} d\vec{S} = 0 \\ \int \vec{D} d\vec{S} = \int \rho dV \\ \int \vec{H} d\vec{\ell} = \int \vec{j} d\vec{S} + \frac{d}{dt} \int \vec{D} d\vec{S} \\ \int \vec{E} d\vec{\ell} = - \frac{d}{dt} \int \vec{B} d\vec{S} \end{cases}$$

$$\begin{cases} \vec{D} = \epsilon \epsilon_0 \vec{E} \\ \vec{B} = \mu_0 \mu \vec{H} \\ \vec{j} = \lambda \vec{E} \\ \frac{d\rho}{dt} + \frac{\partial \rho}{\partial t} = 0 \end{cases}$$

$$P = \int \vec{H} d\vec{S}, \quad \vec{H} = [\vec{E}, \vec{H}]$$

$$J = \chi H \quad \text{магнит. восприимчив.}$$

$$B = \mu H \quad \text{магн. прониц.$$

$$D = \epsilon_0 E + P$$

вектор смещения

$$\vec{D} = \epsilon_0 \epsilon \vec{E}$$

$$\vec{B} = \mu_0 \mu \vec{H}$$

$$\vec{j} = \lambda \vec{E},$$

$$P = \int_S \vec{P} d\vec{S} \quad \{ \vec{E}, \vec{H} \}$$

$$M = I \int_L r \times (dl \times B) \quad \text{— момент сил}$$

$$M = [\int \rho_m \times B] \times B \quad \text{— уг. скорость.}$$

$$\text{Сенсор } \approx 282$$

$$\operatorname{div} D = \int 143 \text{ ед. } 69 \text{ см.}$$

$$\text{Konyak } 69, \text{ стр. } 51$$

$$72, \text{ стр. } 58$$

$$75, \text{ стр. } 60$$

$$\mu_{L_1} \mu_{L_2} = i, \text{ стр. } 273$$

$$W = \int_V \frac{\rho \Delta H}{2} dv, \text{ стр. } 155$$

$$\vec{H} = [\vec{E}, \vec{H}]$$

$$P = \int_S \vec{P} d\vec{S} \quad \text{— энергия}$$

регенерации

$$U_R = IR$$

$$U_C = q/C$$

$$U_L = Li$$

$$U_E = \varepsilon$$

$$U_R = IR$$

$$U_C = \frac{I_0}{i\omega C}$$

$$U_L = I_0 i\omega L$$

$$U_E = \varepsilon$$